Exploring the Possibility of Detecting Tsunami Waves using Partial Data

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Abstract - A tsunami is a series of water waves caused by many factors like earthquakes, volcanic eruptions, landslides and meteorite impacts. This study involves the identification of tsunami waves using partial data before it reaches the shoreline that can be applied as a forecasting tool. The results of the study were based on a discrete-event simulation of a prototype tsunami wave in Deep Ocean and real data of a tsunami event in 2003. Fast Fourier Transform (FFT) was the mathematical tool, which was used in the study. Truncation error occurred when a limited portion of the prototype wave was observed. The exponential function tapered window played a major role in reducing this truncation error. The results showed that when 25% of the prototype tsunami wave started to emerge, it is possible to identify that the wave was of tsunami type and when 37% of the wave starts to emerge it can be verified that the wave is a tsunami wave. The validity of this result was checked by real data of a tsunami event occurred on 17 November 2003. It showed that when nearly 25% of the wave was generated, it could be identified that the wave was of tsunami type. Finally, a tsunami wave superposed with a ship wave in Deep Ocean showed that they have similar patterns and it was not difficult to interpret that the superposed wave was tsunami. The results obtained from this study can be used to forecast whether a tsunami wave is generated. Henceforth, the responsible authorities can take necessary actions to evacuate the residents to safe areas and to reduce the number of false alerts.

Keywords - Tsunami, Fourier transforms, Discrete Fourier transforms, Amplitude spectrum.

1. INTRODUCTION

After the catastrophic tsunami event that occurred on December 26, 2004, people all around the world got to aware of the word 'tsunami' meaning 'harbor wave'. Although there are many causes of tsunami like earthquakes, landslides, volcanic eruptions, meteorite impacts, the most frequent generating events are by earthquakes. Due to this, the main Tsunami Early Warning Systems (TEWS) operating around the world generally rely on observing earthquakes that have more potential to cause tsunamis in order to issue sudden warnings.

The Bureau of Meteorology (BOM) uses Deep Ocean tsunami detection buoys to confirm the existence of tsunamis generated by undersea earthquakes. Pressure variations recorded in Deep Ocean detection systems enable the ability to identify tsunami waves [1]. Therefore, this enhances the capability for early detection and real-time reporting of tsunamis before they reach the land.

2. THEORETICAL BACKGROUND

2.1 Fourier Transforms

Fourier transforms (FT) is a mathematical tool that breaks any function in time domain (or spatial domain) to a function in frequency domain (or wave number domain) characterized by sine and cosine terms [2].

The FT of a continuous-distance signal *f(x)* may be defined as:

$$F(f) = \int_{-\infty}^{+\infty} f(x) e^{-2\pi i f x}\, dx$$

The inverse Fourier transform is defined as:

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(f) e^{2\pi i f x}\, df$$

where x represents distance variable in *kilometers* and f represents *frequency* variable in *radians/kilometers*.

2.2 Discrete Fourier Transforms

Discrete Fourier transforms (DFT) is used for discrete signal because real world problems always deal with finite duration signals that are discretely sampled [2].

The forward and inverse definitions of DFT are:

$$X_k = \sum_{j=0}^{N-1} x_j e^{-\frac{2\pi i j k}{N}}$$

$$j = 0, 1, \ldots, N-1$$

$$x_j = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{\frac{2\pi i j k}{N}}$$

$$k = 0, 1, \ldots, N-1$$

Fast Fourier transform (FFT) is a very fast algorithm that computes DFT. FFT can be used to obtain the amplitude spectrum of a signal in time domain since it is a very fast algorithm that computes DFT in a fraction of time [3].

2.3 Exponential Function

An exponential function with base b is a function of the form:

$$f(x) = b^x$$

where $b > 0, b \neq 1$ is a real number.

An exponential function is always positive. If $0 < b < 1$, $f(x)$ decreases as x increases and if $b > 1$, $f(x)$ increases as x increases.

The exponential function defined by;

$$f(x) = e^x$$

is called the natural exponential function.

3. METHODOLOGY

This paper discusses about the possibility of identifying a tsunami wave using pressure variations before the wave completely traverse over the detection system (i.e. detection of a tsunami wave using partial data). Next, it analyzes the minimum amount of information required to identify a tsunami wave since early detection of a tsunami is useful for giving warnings and alerts. The initial tsunami amplitude can vary according to the magnitude of the earthquake. Therefore, it examines whether it is possible to identify a tsunami wave when it is larger or smaller than the prototype tsunami wave. Ship-generated waves are another type of waves that can occur in Deep Ocean. This paper further discusses about the possibility of identifying a tsunami wave, when a tsunami wave and a ship travel over the detection system simultaneously. Filter/ window functions help to reduce the truncation error effect to a considerable level [2].

The analysis was based on comparing the amplitude spectrum of a prototype tsunami wave with the amplitude spectrums obtained from each of the cases that are going to be examined. In order for comparison, the ratios among each of the spectrums were obtained and analyzed whether the ratios are equivalent to a constant. Hence, it could be interpreted that the prototype tsunami wave pattern is similar to the waves obtained from each of the above cases.

4. SIMULATION/EXPERIMENTAL RESULTS

A prototype tsunami wave and a ship-generated wave were used in order to collect data for the study. The simulation results from MATLAB can be seen in the following figures.

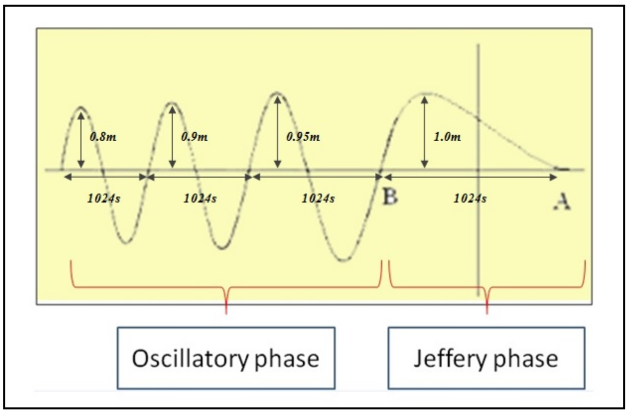

Fig.1. Prototype tsunami wave.

Source: Tantrigoda 2005 [4]

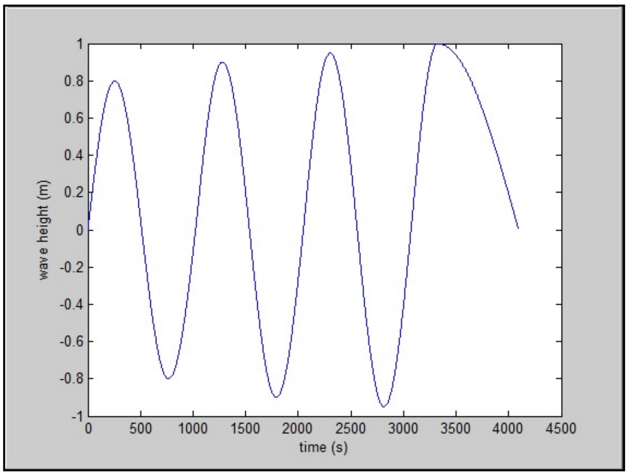

Fig.2. Simulated tsunami wave

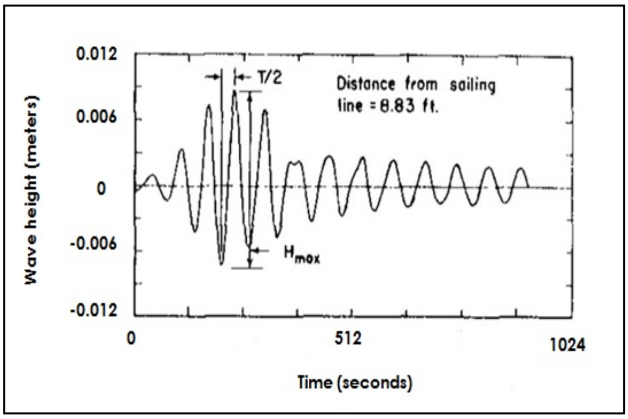

Fig.3. Prototype ship-generated wave
Source: Das 1969 [5]

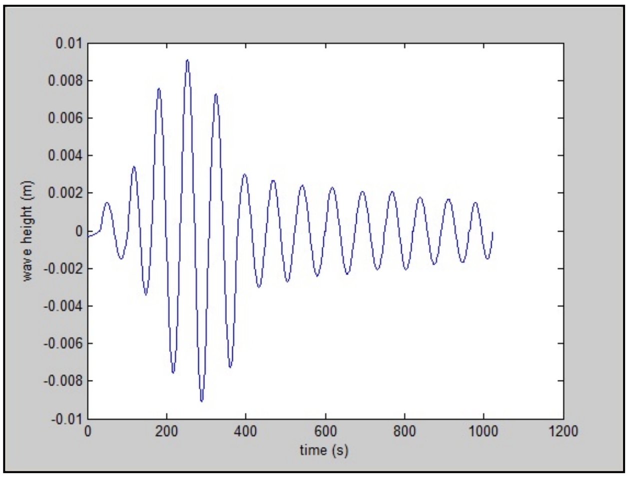

Fig.4. Simulated ship-generated wave

Results

The amplitude spectrums obtained from each of the cases that are going to be examined were compared with the prototype wave. The results are presented under each case.

Case 1:

The prototype tsunami wave was truncated by 75% without the exponential function and with the exponential function. The relevant graphs can be shown below.

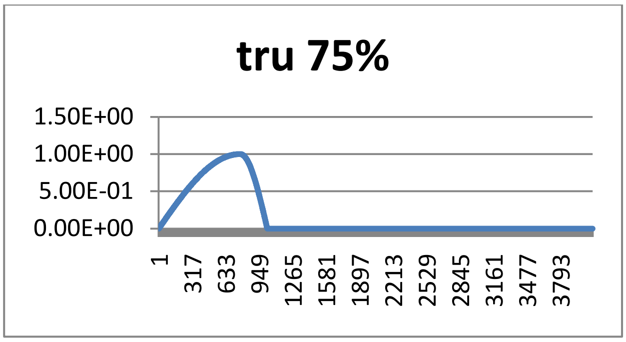

Fig.5. 75% truncated tsunami wave

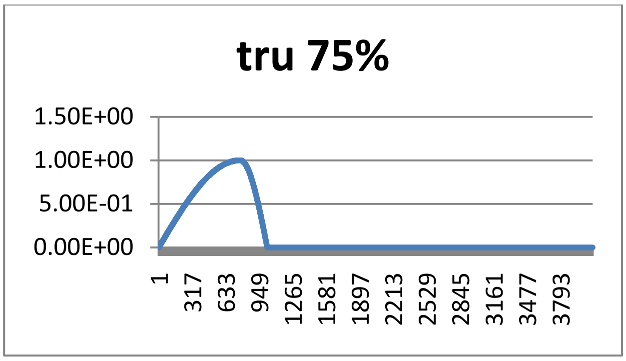

Fig.6. 75% truncated tsunami wave with exponential function

The prototype tsunami wave was truncated by 68% without the exponential function and with the exponential function. The relevant graphs can be shown below.

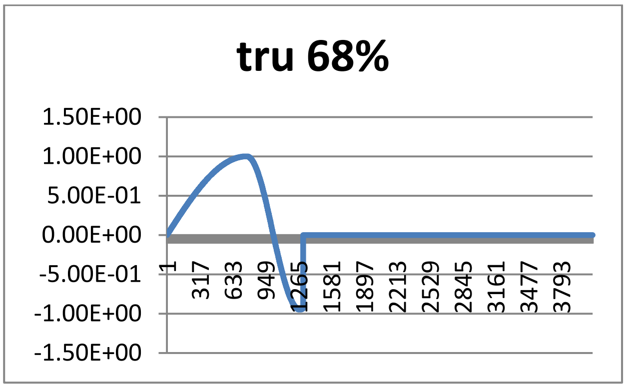

Fig.7. 68% truncated tsunami wave

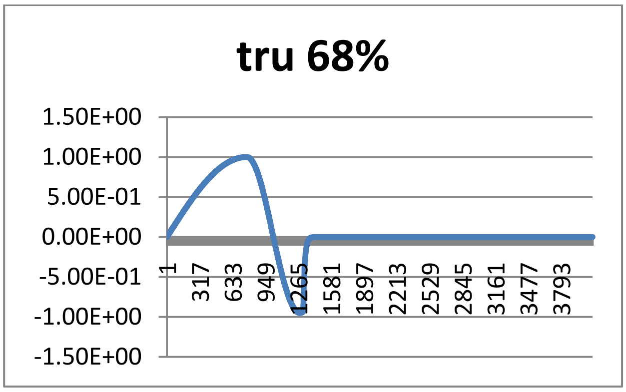

Fig.8. 68% truncated tsunami wave with exponential function

The prototype tsunami wave was truncated by 63% without the exponential function and with the exponential function. The relevant graphs can be shown below.

The ratios among the amplitude spectrums showed that when the truncation percentage is gradually decreasing, the truncated waves at 75% and 63% behaves in a similar manner to the prototype tsunami wave. *i.e.* when 25% and 37% of the prototype wave starts to emerge, they have similar patterns to the prototype wave. Furthermore, when the exponential function was used the truncated wave behaves more closely to the original wave pattern.

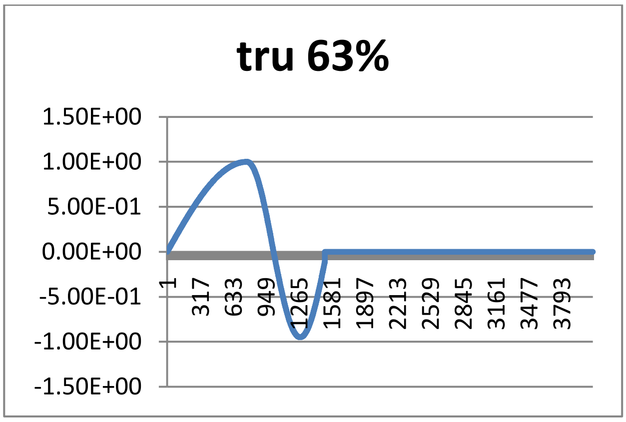

Fig.9. 63% truncated tsunami wave

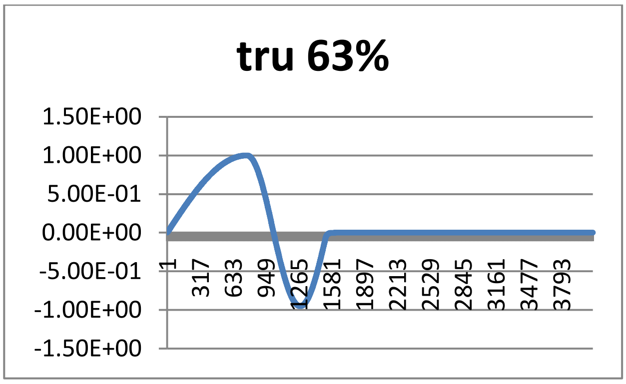

Fig.10. 63% truncated tsunami wave with exponential function

Case 2:

The wave heights of prototype tsunami wave was increased by 66.67% and decreased by 44.44% and the amplitude spectrums were obtained.

When the amplitude spectrum of the prototype tsunami wave was compared with the amplitude spectrum of the tsunami wave of larger amplitude and smaller amplitude, the ratios among them were also increased by 66.67% and decreased by 44.44% respectively.

Case 3:

The validity of the result obtained in Case 1 was examined by real data of a tsunami event which occurred on 17 November 2003 in Alaska.

It showed that when nearly 25% of the tsunami wave was generated, that portion of the wave behaves in a similar manner to that of the original wave pattern.

Case 4:

In order to examine whether there is an effect from the ship-generated waves to tsunami waves, the ship wave was linearly superposed with the tsunami wave. Then the amplitude spectrums of the prototype tsunami wave and superposed wave behaved in a similar manner.

5. CONCLUSION

Based on the results, it is concluded that when nearly 25% of the tsunami wave is visible, it could be identified that the wave is of tsunami type and when 37% starts to emerge it could be verified that the wave is a tsunami wave. When a tsunami wave probably of larger amplitude than the prototype wave or smaller than the prototype wave is generated, it is possible to identify it. The real data of tsunami event on 17 November 2003 showed that when 25% of the wave was generated, it is evident that a tsunami wave had been occurred. The effect from a ship wave that, traverses over the detection system at the same time a tsunami wave is generated showed that there is no effect from the ship-generated waves to the tsunami wave. Hence, it is possible to identify a tsunami wave passes through the detection system although a ship traverses over it.

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ACKNOWLEDGEMENT

Authors wish to thank Prof. D.A. Tantrigoda for originally suggesting this problem and for many useful discussions.

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