

# Ordered QR Decomposition Aided Multiple Feedback Successive Interference Cancellation Algorithm for Large MIMO Detection

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**Abstract** - Multiple antenna systems are gaining an increasing interest for future generation of wireless communication systems. Multiple input multiple output (MIMO) systems are capable of achieving linear increase in the channel capacity with respect to minimum of the number of transmit and the receive antennas. However, one of the major challenges in MIMO systems is the problem of reliable symbol detection at the receiver end. In this paper, we present an log-likelihood ratio ordered QR decomposition aided multiple feedback successive interference cancellation (LLR-QR-MFSIC) for symbol detection in large MIMO systems. In multiple feedback (MF), multiple constellation points are used in the decision feedback loop of successive interference cancellation (SIC) based MIMO detector. In our work, we incorporate QR decomposition based SIC and use the MF strategy to improve the bit error rate (BER performance). But, QR aided SIC suffer from the error propagation due to wrong decisions in early layers, and therefore, the BER performance degrades. Hence, in our work we propose to use LLR based ordering for QR aided MF SIC algorithm which successfully mitigates the error propagation by ordering the detection sequence. Through simulations we confirm the advantage of ordering when compared with the QR aided MF SIC without ordering.

**Keywords** – MIMO, Multiple-Input-Multiple-Output, SIC, Successive Interference Cancellation, BER, Bit Error Rate, QAM, Quadrature Amplitude Modulation.

## 1. INTRODUCTION

With the rapid advance in technology, the demand for data traffic in wireless systems is increasing exponentially. To overcome the throughput requirement of all the users, several new techniques have been evolved such as long term evolution (LTE), LTE advanced (LTE-A) and wireless interoperability for microwave access (WiMAX), and a new generation of wireless communications is developed known as 4th generation (4G) and 5th generation (5G). Multiple-input multiple-output (MIMO) is being adopted as a promising technique for enhancing the capacity of wireless channel in 4G/5G wireless standards [1]. The increase in capacity is proportional to the minimum of the transmit and the receive antennas in MIMO systems. MIMO systems with tens to hundreds of antennas is said to be large MIMO systems. Through

spatial multiplexing multiple symbols can be transmitted simultaneously from different transmit antennas. However, reliable detection of these multiple symbols at the receiver end is non-deterministic polynomial (NP) hard. Multiple antenna systems have been promising in terms of providing diversity and multiplexing gains in wireless channel [2], [3]. Maximum likelihood (ML) detection in MIMO systems is used to achieve the optimal performance by jointly detecting the symbol vector and is known to achieve minimum bit error rate (BER) performance. ML detector performs exhaustive search over all the possible transmit vectors and finally select the one which is best amongst all the solutions. The number of possible transmit vectors increases exponentially with respect to the number of transmit antennas or the modulation order, and therefore, the computational complexity of ML detector is exponential [4]. Practical implementation of ML detector is impractical due to its high computational complexity. There are several low complexity MIMO detectors proposed in the literature. These include zero forcing (ZF) detector, minimum mean squared error (MMSE) detector and successive interference cancellation (SIC) detector [5]. Another such detector is sphere decoder (SD) which achieves a near MLD performance with reduced complexity compared to the MLD [6]. Several other algorithms in this context have been proposed in the literature which includes [7]–[18].

QR decomposition aided successive interference cancellation (SIC) is a well-known technique which is used for sequential detection of symbols by decomposing the MIMO channel into an orthonormal matrix  $Q$  and an upper triangular matrix  $R$  [7], [8]. SIC is also known as layered detection where symbol corresponding to each transmit antenna is detected in a layer. However, SIC based MIMO detector suffers from error propagation and hence, its BER performance degrades. Therefore, to overcome this problem, a multiple feedback scheme is used where the concept of shadow region is used in the decision feedback loop [9]. In multiple feedback strategy, multiple constellation points are used in the decision feedback loop of the SIC detector whenever a shadow condition occurs. The shadow condition occurs if the distance between the

estimated value of the decision and the nearest constellation point is above a finite threshold value which means that the decision is unreliable.

Alternatively, in our work we integrate the concept of multiple stages with the QR decomposition aided SIC. This improves the decision accuracy in each layer. Further, to mitigate the error propagation, we use log likelihood ratio based ordering for ordering the detection sequence. Through simulations we corroborate the superiority of the proposed algorithm over the ordered SIC detector. Further, the BER performance of our work improves with increase in the number of transmit antennas which is very useful for systems with large number of antennas i.e. large-MIMO systems.

The rest of the paper is organized as follows. In section 2, we present the mathematical model of the point to point MIMO system used in the paper. In section 3, the traditional MIMO detectors such as maximum likelihood detection, zero forcing detectors, minimum mean squared error detector and QR aided SIC detector. The proposed method is discussed in section 4. In section 5, the simulation results on BER performance are discussed for 12 \_ 12; 16 \_ 16 and 32 \_ 32 MIMO systems with 4-QAM signalling. Finally in section 6, we conclude the article.

## 2. SYSTEM MODEL

In this article, a MIMO system with  $N$  transmit antennas and  $M$  receive antennas is considered. Let  $\mathbf{x} = [x_1, x_2, \dots, x_N]^T$  represent the transmit vector with symbol  $x_j$  being transmitted from the  $j^{\text{th}}$  transmit antenna. The symbol  $x_i$  is taken from a constellation set represented by  $\mathcal{C}$  (for e.g. BPSK, QPSK and 4-QAM). The channel through which the symbols are transmitted is given by  $\mathbf{H}$ , and is assumed to be a Rayleigh flat fading channel. The receiver receives a modified vector (i.e. through channel variations and receiver noise)  $\mathbf{y}$  as given by

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (1)$$

where the channel matrix  $\mathbf{H}$  is considered as an  $M \times N$  MIMO channel matrix with each element  $h_{kl}$  for  $k = 1, 2, \dots, M$  and  $l = 1, 2, \dots, N$  represents the channel gain between the  $l^{\text{th}}$  transmit antenna and the  $k^{\text{th}}$  receive antenna. The vector  $\mathbf{n}$  is additive white Gaussian noise (AWGN) vector and each element  $n_i$  for  $i = 1, 2, \dots, N$  independent and identically distributed (i.i.d.) and  $\sim CN(0, \sigma^2)$ ,  $\sigma^2$  is the noise variance.

$$\mathbf{r} = \mathbf{G}\mathbf{s} + \mathbf{w} \quad (2)$$

where  $\mathbf{r} = \begin{bmatrix} \Re(\bar{\mathbf{y}}) \\ \Im(\bar{\mathbf{y}}) \end{bmatrix}_{2M \times 1}$ ,  $\mathbf{G} = \begin{bmatrix} \Re(\bar{\mathbf{H}}) & -\Im(\bar{\mathbf{H}}) \\ \Im(\bar{\mathbf{H}}) & \Re(\bar{\mathbf{H}}) \end{bmatrix}_{2M \times 2N}$ ,  $\mathbf{s} = \begin{bmatrix} \Re(\bar{\mathbf{x}}) \\ \Im(\bar{\mathbf{x}}) \end{bmatrix}_{2N \times 1}$ , and  $\mathbf{w} = \begin{bmatrix} \Re(\bar{\mathbf{n}}) \\ \Im(\bar{\mathbf{n}}) \end{bmatrix}_{2M \times 1}$ .  $\Re(\cdot)$  and  $\Im(\cdot)$  denote the real and imaginary parts of  $(\cdot)$  respectively. The constellation set with real entries is  $\mathcal{O}$ .

## 3. TRADITIONAL MIMO DETECTORS

In this section, we present overview of traditional MIMO detectors such as maximum likelihood detector (MLD), zero forcing (ZF) detector, minimum mean squared error (MMSE) detector and QR aided successive interference cancellation (SIC) detector.

### A. Maximum Likelihood detector

The maximum likelihood detection is known to achieve the optimal (minimum) bit error rate performance. In ML, an exhaustive search is performed over all the possible transmit vectors. The ML solution is given by

$$\mathbf{s}_{ML} = \arg \min_{\mathbf{s} \in \mathcal{O}^N} \|\mathbf{r} - \mathbf{G}\mathbf{s}\|^2 \quad (3)$$

where  $\mathcal{O}^N$  denotes the set of all possible symbol vectors which could be transmitted by the transmitter. But, the main drawback of MLD is that the number of computations required for performing the exhaustive search grows exponentially with increase in the number of transmit antennas due to which MLD becomes impractical.

### B. Zero forcing detector

The zero forcing (ZF) detection is a linear detection method for MIMO systems. In ZF, a transformation matrix is used which removes the effect of channel gain matrix from the received vector by multiplying it with the pseudo inverse matrix as

$$\mathbf{T}_{ZF} = (\mathbf{G}^G \mathbf{G})^{-1} \mathbf{G}^H \quad (4)$$

where  $(\cdot)^H$  denote the Hermitian transpose of the matrix and  $(\cdot)^{-1}$  denotes the inverse of a matrix. This matrix is then multiplied with the received vector as.

$$\mathbf{z}_{ZF} = \mathbf{T}_{ZF}\mathbf{r} \quad (5)$$

$$\mathbf{z}_{ZF} = \mathbf{s} + \mathbf{T}_{ZF}\mathbf{w} \quad (6)$$

Now every entry of the estimated vector is quantized to the nearest constellation point as

$$s_{ZF}(i) = Q[z_{ZF}(i)] \quad \forall i = 1, 2, \dots, N \quad (7)$$

where  $Q[\cdot]$  is the quantization operator.

### C. Minimum mean squared error detector

In minimum mean squared error (MMSE) detection technique, the mean squared distance between the transmitted vector and the estimated vector is minimized and the transformation matrix  $\mathbf{T}_{MMSE}$  is found which is given by

$$\mathbf{T}_{MMSE} = (\mathbf{H}^H \mathbf{H} + \sigma^2 \mathbf{I}_{NT})^{-1} \mathbf{H}^H \quad (8)$$

Similar to the ZF detection, in MMSE also the transformation matrix is multiplied with the received vector followed by the quantization.

### D. Successive interference cancellation detector

In QR aided successive interference cancellation (SIC), the symbols are detected sequentially for corresponding to each transmit antenna by using the QR decomposition of the channel matrix  $\mathbf{H}$ . After every successful decision about a symbol, its interference from the received vector is cancelled and the decision about next symbol in the sequence

is taken accordingly. The steps involved in the SIC detection technique are shown in Algorithm 1.

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#### Algorithm 1 QR aided SIC algorithm for MIMO detection

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- 1: **input:**  $\mathbf{r}, \mathbf{G}, N, M$ ;
  - 2: Compute  $\tilde{\mathbf{r}} = \mathbf{Q}^H \mathbf{r}$  where  $\mathbf{G} = \mathbf{QR}$
  - 3: **for**  $k = 2N : -1 : 1$  **do**
  - 4:  $\hat{s}_k = \frac{\tilde{r}_k - \sum_{j=k+1}^{2N} R_{k,j} \tilde{s}_j}{R_{k,k}}$
  - 5:  $\tilde{s}_k = Q(\hat{s}_k)$
  - 6: **end for**
  - 7: **output:**  $\tilde{\mathbf{s}} = [\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_{2N}]^T$  is output solution vector
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## 5. PROPOSED DETECTION SCHEME

In this section, we present the proposed algorithm for symbol detection in MIMO systems. In the proposed work we incorporate the log likelihood ratio (LLR) based ordering for ordering the detection sequence of SIC algorithm. The LLR based ordering is performed while decomposing the channel matrix  $\mathbf{G}$  into an orthonormal

matrix  $\mathbf{Q}$  and an upper triangular matrix  $\mathbf{R}$ . The algorithm for LLR ordered QR decomposition is shown in Algorithm 2.

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#### Algorithm 2 LLR-based ordering Algorithm for QR Decomposition

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- 1: **initialize:**  $\mathbf{R} = \mathbf{0}, \mathbf{Q} = \mathbf{G}, \Phi = (1, \dots, 2N)$ ;
  - 2: **for**  $l = 1, \dots, 2N$  **do**
  - 3:  $m_l = \arg \min_{i=l, \dots, 2N} (\frac{|\mathbf{q}_i^H \mathbf{r}|}{\|\mathbf{q}_i\|})$ ;
  - 4: where  $\mathbf{q}_i$  is the  $i^{\text{th}}$  column of matrix  $\mathbf{Q}$
  - 5: interchange columns  $l$  and  $m_l$  in  $\mathbf{Q}, \mathbf{R}, \Phi$ ;
  - 6:  $r_{l,l} = \|\mathbf{q}_i\|$ ;
  - 7:  $\mathbf{q}_l = \frac{\mathbf{q}_i}{r_{l,l}}$ ;
  - 8: **for**  $i = l + 1, \dots, 2N$  **do**
  - 9:  $r_{l,i} = \mathbf{q}_l^H \mathbf{q}_i$ ;
  - 10:  $\mathbf{q}_i = \mathbf{q}_i - r_{l,i} \mathbf{q}_l$ ;
  - 11: **end for**
  - 12: **end for**
- 

Once the ordered QR decomposition is done, the matrices  $\mathbf{Q}$  and  $\mathbf{R}$  are fed to the SIC based detector for detection. However, the SIC based detector as shown in Algorithm 1 is modified by integrating the concept of multiple feedback strategy. Therefore, after every decision its reliability is checked by comparing the distance between the soft value of the decision and the nearest constellation point. If the distance is more than the predefined threshold value (say  $d$ ) then the decision is said to be unreliable and multiple neighbouring constellation points are used in the decision feedback loop followed by the SIC based detector. If the distance is less than  $d$  then the decision is said to be reliable and the detection procedure is continued for the remaining layers. The steps involved in the QR aided ordered multiple feedback SIC algorithm are as below

*Step 1:* Perform LLR based QR decomposition of the channel matrix  $\mathbf{G}$  as shown in Algorithm 2.

*Step 2:* Initiate the QR aided SIC as shown in Algorithm 1.

*Step 3:* Compute  $dis = |\tilde{s}_k - Q(\hat{s}_k)|$  and check if  $dis \leq d$

then the decision is reliable and declare  $\tilde{s}_k = Q(\hat{s}_k)$  else if  $dis > d$  then the decision falls in the shadow region and is declared unreliable and follow the multiple feedback criteria according to [10].

*Step 4:* Declare the final result and terminate the procedure.

## 6. SIMULATION RESULTS

In this section, the simulation results on bit error rate versus signal to noise ratio are compared for the ordered SIC and the proposed algorithm with different threshold radius values such as  $d = 0.1, 0.2, 0.5$ . The simulations are performed in MATLAB software. For comparing the BER, we considered  $12 \times 12, 16 \times 16$  and  $32 \times 32$  MIMO systems with 4-QAM signalling.

In figure 1, the BER performance for  $12 \times 12$  MIMO systems is plotted with respect to the SNR. It is observed that the BER performance of the proposed algorithm is superior over the ordered SIC algorithm. Also, the performance of the proposed algorithm improves with increase in the threshold radius  $d$  from 0.1 to 0.5. Similarly, in figure 2 and 3, the BER performance is simulated for  $16 \times 16$  and  $32 \times 32$  MIMO systems with 4-QAM, respectively. The observations are also similar to that of the figure 1. In figure 4, the BER performance for MIMO systems with different number of antennas is compared for the proposed algorithm. It is observed that the BER performance of the proposed algorithm improves with increase in number of antennas which is an additional benefit when number of antennas are large i.e. large-MIMO systems

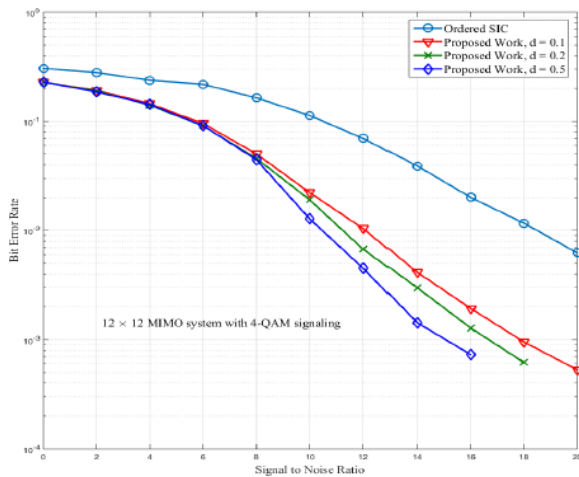


Fig. 1. Bit error rate performance of  $12 \times 12$  multiple-input multiple-output system versus signal to noise ratio (dB)

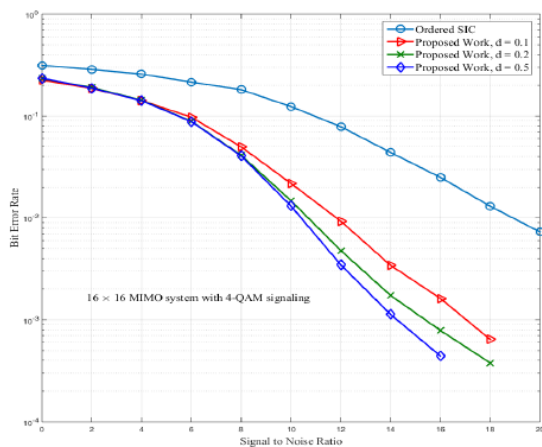


Fig. 2. Bit error rate performance of  $16 \times 16$  multiple-input multiple-output system versus signal to noise ratio (dB)

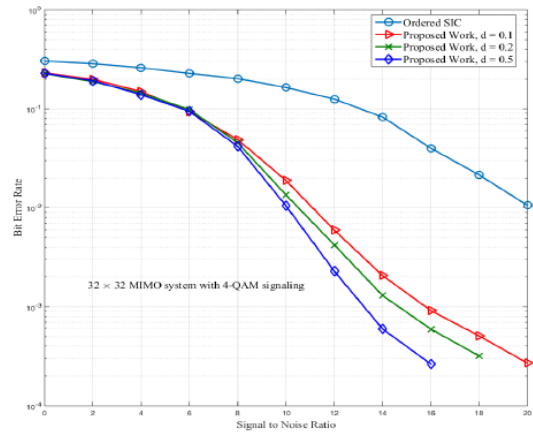


Fig. 3. Bit error rate performance of  $32 \times 32$  multiple-input multiple-output system versus signal to noise ratio (dB)

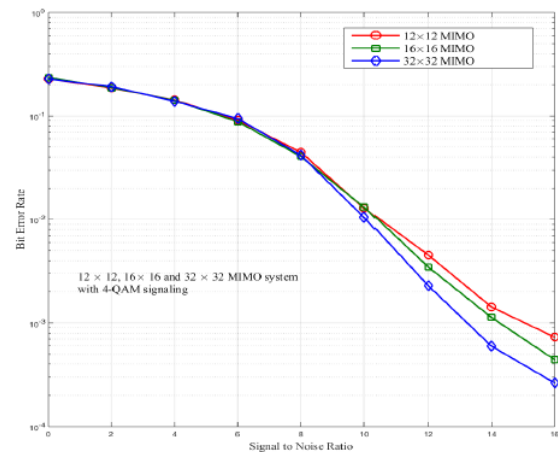


Fig. 4. Bit error rate performance of different multiple-input multiple-output system versus signal to noise ratio (dB)

## 7. CONCLUSION

In this paper, a QR decomposition aided LLR ordered multiple feedback successive interference cancellation algorithm is proposed for symbol detection in large MIMO systems. The underlying concept of using the LLR ordering is to consider the instantaneous noise along with the signal power for ordering the detection sequence. The multiple feedback n SIC is used to improve the accuracy of decisions thereby an enhanced BER performance can be achieved. Through simulation results, the BER performance of the proposed algorithm for different MIMO systems is compared with the ordered SIC and it is observed that the BER performance of the proposed algorithm is superior and improves with increases with increase in the threshold radius value. Furthermore, the BER performance of the proposed algorithm improves with increase in the number of transmit antennas, and hence, the algorithm is very well suited for systems with large number of antennas i.e. large-MIMO systems.

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