

## REVIEW ARTICLE

# Study of Different Transformation Optics Method (TOM) for Patch Antenna Design

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## ABSTRACT

The theme of this paper is a stimulating research topic titled transformation optics (TO) and its application as a regulator of the flow of electromagnetic waves in association with the concept of metamaterial. Although the fundamental principle of deformation of wave path in an inhomogeneous medium has been known for decades, it was only in 2006 that the concept of transformation optics was established to materialize space deformation to give light such a desired path. Such a concept is, then, able to allow the design of novel and unimaginable electromagnetic and optical devices for various functionalities. In this effort, we look at the transformation optics beyond its guidance of light paths. With different appropriate examples, we establish that transformation optics can be used to change electromagnetic fields up to an unparalleled level. This manuscript focuses on the theoretical tools defining the transformation optics concept. We address the antenna applications using transformation optics to provide the readers with the entire process of designing a device based on transformation optics.

## KEYWORDS

Transformation Optics Method (TOM), planar luneburg lens, high impedance surface, Multiband, Metamaterial.

## 1. INTRODUCTION

Transformation optics (TO) is an emerging technique for the design of advanced electromagnetic (EM) media. It is based on the concept that Maxwell's equations can be written in a form-invariant manner under coordinate transformations, such that only the permittivity and permeability tensors are modified. It enables altering the field distribution by applying material parameters which cause light to "behave" as if it was in a transformed set of coordinates. Moreover, the combination of TO and structured metamaterials allows for the design and realization of material properties generated by the TO concept. The circle and the square are among the most common shapes used by humans, it is certainly worthwhile to study the mathematical correspondence between the two. In this paper, we shall discuss ways of transformation optics to map a region into another, particularly a circular shape into a square shape and vice-versa. There are extremely many ways of doing this mapping but we focused on mappings with nice closed-form invertible equations. We emphasize the importance of invertible equations because we want to perform the mapping back and forth between the circular disc and the square. We shall present and discuss different such transformation of mappings in the subsequent section. The geometric

interpretation of Maxwell's equations utilized in the TO approach provides a powerful and intuitive design tool for the manipulation of EM fields on all frequency range. The TO technique, however, extends well beyond the domain of computational approaches and has gained a great deal of relevance over the past decade in conjunction with the emerging field of metamaterials. Metamaterials are artificially structured media whose effective material parameters can be engineered to have, in principle, any combination of anisotropic electric and magnetic responses, making them an enabling path for transformation optical structures.

The rest of the paper is organized in the following sections section II describe the overview of transformation optics and different type of mapping particularly from circular to square and vice versa along with the equation involved in these transformations. In section III we will discuss the latest research in the field of transformation optics especially for 3d Luneburg lens antenna (LLA) into 2d thin flat microstrip patch antenna with unchanged performance using transformation optics method (TOM) designing. In section IV we conclude the present era research and based on that will identify our basic design method and present future claims for our research work.

## 2. THEORY OF TRANSFORMATION OPTICS

The physical meaning of transformation optics can be understood with the help of a cartesian system with a predefined value of electric and magnetic fields along with the corresponding Poynting vectors. Now, imagine that the coordinates are uninterruptedly distorted into another coordinate system. Transformation optics was brought into the creation with the fact that as the structure is distorted it changes all the associated fields with them. Hence, to direct the path of EM wave, only distortion in the primary coordinate system is required. Knowledge of the transformation optics provides the values of permittivity and permeability ( $\epsilon$  and  $\mu$ ) required to direct the electromagnetic wave.

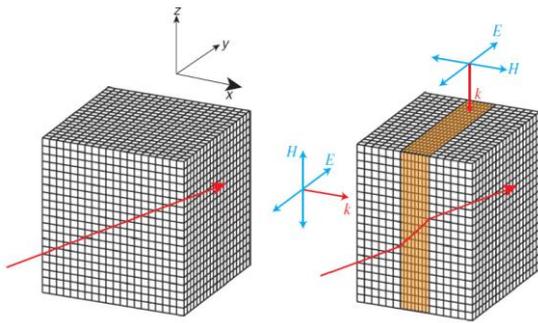


Fig 1. Compresses coordinate transformation along the x-axis.

Figure 1 shows a general follows of EM wave over two different paths with the help of a red line which appears the same when compared with the compressed part in terms of direction and phase as before. We can forecast the metamaterial characteristics of the shaded region (brown) that would realize the above flow of EM wave. In the fig. 1 the EM wave is propagating along the x axis, the condition  $k_0d = k'ad$  is required for the phase conservation at the far side of the compressed region. Here,  $k'$  is the wave vector in the compressed region and  $k_0$  is the free-space wave vector where  $a$  is compression factor,  $d$  is the thickness of uncompressed layer. This above condition requires  $k' = k_0\sqrt{\epsilon'_y\mu'_z}$  thus  $k_0d = k_0\alpha d\sqrt{\epsilon'_y\mu'_z}$  and hence  $1/a = \sqrt{\epsilon'_y\mu'_z}$ . Now due the presence of symmetry between magnetic and electric field the value of  $\epsilon$  and  $\mu$  appear on the same in the transformed space. Because of the no compression in Z direction i.e the direction of wave propagating, the value of refractive index is unchanged and hence,

$$\sqrt{\epsilon'_y\mu'_z} = \sqrt{\epsilon_y\mu_z} = 1$$

Therefore  $\epsilon'_x = \mu'_x = \alpha$  this is because of the fact that Maxwell's equations are symmetrical for  $\epsilon$  and  $\mu$  and  $\epsilon'_y = \mu'_y = \epsilon'_z = \mu'_z = \alpha^{-1}$  in short, we can say that the value of if we compress a coordinate system along a certain axis, both  $\epsilon$  and  $\mu$  are decreased by the compression factor in the direction of distortion. On the other hand,  $\epsilon$  and  $\mu$  are

increased by the inverse of the compression factor in the direction perpendicular to that of the distortion. This approach of transformation is very useful in patch antenna designing by the help of metamaterials, in general the distortion of EM wave depends on the transformation of the electric permittivity and magnetic permeability tensors

$$\bar{\epsilon}' = \frac{\bar{\Delta}\bar{\epsilon}\bar{\Delta}^T}{\det\bar{\Delta}}, \bar{\mu}' = \frac{\bar{\Delta}\bar{\mu}\bar{\Delta}^T}{\det\bar{\Delta}} \quad (1)$$

Where equation (1) represents permittivity and permeability tensor ( $\bar{\epsilon}'$ ,  $\bar{\mu}'$ ) of physical space ( $x', y', z'$ ) corresponding to virtual system ( $x, y, z$ ) and  $\bar{\Delta}$  is the jacobian matrix and can be expressed as,

$$\bar{\Delta} = \begin{bmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} & \frac{\partial x'}{\partial z} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} & \frac{\partial y'}{\partial z} \\ \frac{\partial z'}{\partial x} & \frac{\partial z'}{\partial y} & \frac{\partial z'}{\partial z} \end{bmatrix}$$

Now we discuss some of the stretching technique for transformation of a circle into a square with the help of signum function [ $sgn(x)$ ].

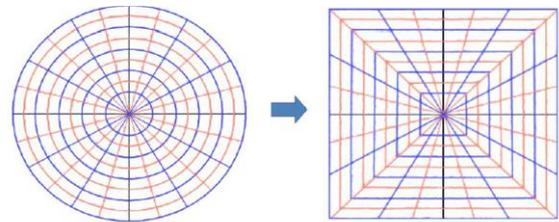


Fig 2. Simple Stretching transformation from circle to square along the x axis.

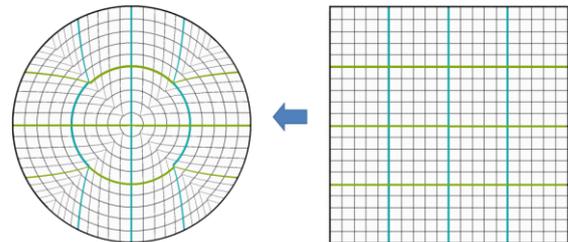


Fig 3. Simple Stretching transformation from square to circle along the x axis.

Equation for circle to square mapping is given by

$$x = \begin{cases} sgn(u)\sqrt{u^2 + v^2} & \text{when } u^2 \geq v^2 \\ sgn(v) \frac{u}{v} \sqrt{u^2 + v^2} & \text{when } u^2 < v^2 \end{cases}$$

$$y = \begin{cases} sgn(u) \frac{v}{u} \sqrt{u^2 + v^2} & \text{when } u^2 \geq v^2 \\ sgn(v) \sqrt{u^2 + v^2} & \text{when } u^2 < v^2 \end{cases}$$

and mapping of square to circle is given by

$$u = \begin{cases} \operatorname{sgn}(x) \frac{x^2}{\sqrt{x^2 + y^2}} & \text{when } x^2 \geq y^2 \\ \operatorname{sgn}(y) \frac{xy}{\sqrt{x^2 + y^2}} & \text{when } x^2 < y^2 \end{cases}$$

$$v = \begin{cases} \operatorname{sgn}(x) \frac{xy}{\sqrt{x^2 + y^2}} & \text{when } x^2 \geq y^2 \\ \operatorname{sgn}(y) \frac{y^2}{\sqrt{x^2 + y^2}} & \text{when } x^2 < y^2 \end{cases}$$

where  $(x,y)$  are physical space and  $(u,v)$  are virtual space . There are various transformations present in different literature, in the next section we will discuss about the latest research in the field of transformation optics.

### 3. LITERATURE REVIEW

In this part we will discuss on the latest research which is going on in present era specially in the field of transformation optics especially for mapping of 3d luneburgs antenna in to 2d printed antennas.

[1] YChen, Zhi Ning Su, Yuanyan Liu, Wei E.I, "Metantennas: Flat luneburg lens antennas using transformation optics method (TOM).", 2020 Int. Work. Antenna Technol. iWAT 2020, pp. 17-19, 2020. In this paper, two types of flat Luneburg lens antenna based on printed circuit board technique for beamscanning applications and achive Single operating band with 20% bandwidth for  $|S_{11}| < 10$  dB. The transformation optics is used to flatten the design of conventional spherical Luneburg lenses and the technology based on metamaterials is proposed to implement the gradient index lens structure.

[2] B. Hu, T. Wu, Y. Cai, W. Zhang, and B. L. Zhang, "A Novel Metamaterial-Based Planar Integrated Luneburg Lens Antenna with Wide Bandwidth and High Gain," *IEEE Access*, vol. 8, pp. 4708-4713, 2020.

In this paper, A novel metamaterial-based and broadband planar integrated Luneburg lens antenna is designed, fabricated, and measured. The bandwidth of 16 to 28 GHz is realized with the return loss below -10 dB.

[3] Y. Su and Z. N. Chen, "A Radial Transformation-Optics Mapping for Flat Ultra-Wide-Angle Dual-Polarized Stacked GRIN MTM Luneburg Lens Antenna," *IEEE Trans. Antennas Propag.*, vol. 67, no. 5, pp. 2961-2970, 2019.

In this paper, Transformation mapping has been proposed to radially compress a spherical Luneburg lens into a cylindrical slab with a small thickness. Here we get Single operating band with 20% bandwidth for  $|S_{11}| < 10$  dB.

[4] G. Cheng, Y. M. Wu, J. X. Yin, N. Zhao, T. Qiang, and X. Lv, "Planar Luneburg Lens Based on the High Impedance

Surface for Effective Ku-Band Wave Focusing," *IEEE Access*, vol. 6, no. 8, pp. 16942-16947, 2018.

In this paper, A planar Ku-band Luneburg lens is designed. to achieve good wave focusing effects in the Ku band and based on the high impedance surface, metamaterial units are designed.

[5] Y. Su and Z. N. Chen, "A flat dual-polarized transformation-optics beamscanning luneburg lens antenna using PCB-Stacked gradient index metamaterials," *IEEE Trans. Antennas Propag.*, vol. 66, no. 10, pp. 5088-5097, 2018. In this paper, based on a transformation optics method, a flat compact dual-polarized Luneburg lens antenna is proposed. Here we get single Band of operation with a center frequency of 10.5 GHz.

[6] A. Sayanskiy, S. Glybovski, V. P. Akimov, D. Filonov, P. Belov, and I. Meshkovskiy, "Broadband 3-D Luneburg Lenses Based on Metamaterials of Radially Diverging Dielectric Rods," *IEEE Antennas Wirel. Propag. Lett.*, vol. 16, no. XX, pp. 1520-1523, 2017.

In this paper, we describe and study microwave Luneburg lenses using a broadband metamaterial composed of radially diverging dielectric rods. Here we use 3-D Luneburg Lenses is used for transmission.

[7] X. Yu, M. Liang, and H. Xin, "Performance evaluation of wideband microwave direction-of-arrival estimation using luneburg lens," *IEEE Antennas Wirel. Propag. Lett.*, vol. 16, pp. 2453-2456, 2017.

In this letter, a wideband passive direction-finding (DF) system utilizing the Luneburg lens is investigated. Here a 3-D Luneburg Lenses is used for transmission with a centre frequency of 10 GHz.

[8] Y. Cai, Z. P. Qian, Y. S. Zhang, J. Jin, and W. Q. Cao, "Bandwidth enhancement of SIW horn antenna loaded with air-via perforated dielectric slab," *IEEE Antennas Wirel. Propag. Lett.*, vol. 13, pp. 571-574, 2014.

A substrate integrated waveguide (SIW) horn antenna loaded with air-via perforated dielectric slab for bandwidth enhancement is proposed in this letter. With the help of 3-D Luneburg Lenses we get the impedance bandwidth of 40% from 16 to 24 GHz.

[9] C. Mateo-Segura, A. Dyke, H. Dyke, S. Haq, and Y. Hao, "Flat luneburg lens via transformation optics for directive antenna applications," *IEEE Trans. Antennas Propag.*, vol. 62, no. 4, pp. 1945-1953, 2014.

In this research work a conventional Luneburg lens is redesigned accounting for dielectric materials that implement a coordinate transformation. Here we get single Band of operation with very narrow beams.

[10] T. Driscoll et al., "Performance of a three-dimensional transformation-optical-flattened Luneburg lens," *Opt. Express*, vol. 20, no. 12, p. 13262, 2012.

In This paper demonstrate both the beam-forming and imaging capabilities of an X-band (8-12 GHz) operational Luneburg lens which give an idea of using metamaterials in the implementation of TO.

#### 4. CONCLUSION AND FUTURE WORK

The proposed designs of this manuscript are organized in different section, first we check the basic theory associated with transformation optics and its effect on permittivity and permeability for EM wave propagation with the help of analytical evidence and established formula after that we examine the transformation of a circular disc into a square shape and vice versa by simple stretching and the whole process is explained mathematically with the help of signum function. In next part we identify the impact of transforming 3d luneburg lens antenna in to 2d printed antenna with the help of different literature available.

With the help of different research work, we can conclude that the transformation optics has great applications in designing of electromagnetic wave-based devices this technique help us in simplifying the complex 3d design into simple 2d structure without compromising with the different performance parameters. The introduction of transformation optics technique improves the bandwidth and gain of different patch antenna, this technique is also very useful in order to get multiband radiation abilities. This work opens the door of further research in bandwidth and gain improvement for metamaterial patch antenna structure.

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