

# Performance Comparison of Non-Linear Detectors for MIMO System in Spatial Multiplexing

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**Abstract** - With the integration of Internet and multimedia applications in next generation wireless communications, the demand for wide-band high data rate communication services is growing. As the available radio spectrum is limited, higher data rates can be achieved only by designing more efficient signaling techniques. Wireless communication using Multiple-Input Multiple-Output (MIMO) links has emerged as one of the most significant breakthrough in modern communication. By employing multiple antennas at transmitter and receiver sides, MIMO techniques enable a new dimension- the spatial dimension - that can be utilized in different ways to combat the impairments of wireless channels. While using MIMO techniques, there is inter-symbol interference present between the symbols. Detection is a well known technique for combating inter-symbol interference. MIMO system consists of mainly two types of detectors: Linear and Non-linear detectors. Non-linear detector outperform than linear detector. Performance of the non-linear detectors like Maximum Likelihood (ML), ZF-SIC (Zero Forcing Successive Interference Cancellation), MMSE-SIC (Minimum Mean Square Error- Successive Interference Cancellation) are compared and analysed for different BER vs Eb/No in spatial multiplexing domain.

**Keywords** - MIMO, ML, ZF-SIC, MMSE-SIC, BER, Eb/No.

## 1. INTRODUCTION

Recently, multiple-input multiple-output (MIMO) wireless systems have attracted considerable attention in the communications community. Such systems employ multiple antennas, or antenna arrays, at both the transmitter and the receiver to enable spatial multiplexing of data and, thus, increased data rates[1]. Traditionally, multiple antennas have been used at the receiver to provide spatial diversity and mitigate the effects of signal fading due to multipath propagation in the channel. However, recent developments in information theory have shown that by using multiple transmit and receive antennas, signal fading can in fact be turned into an advantage[2]. With multiple antennas at both the transmitter and the receiver, spatially distributed channels can be supported simultaneously in the same frequency band, and by transmitting data in parallel through these channels the data rate can be increased [4]. When deployed in a rich scattering environment, such systems are capable of greatly

increasing the spectral efficiency over traditional single channel systems.

The capacity of the flat MIMO Rayleigh fading channel associated with a system with  $N$  transmit antennas and  $M \geq N$  receive antennas is given as[4][6]

$$C = \log_2(\det[\mathbf{I}_M + \rho \mathbf{H}\mathbf{H}']) \text{ bit/s/Hz}, \quad (1)$$

Where  $\mathbf{I}_M$  is the  $M \times M$  identity matrix,  $\rho$  is the signal-to-noise ratio (SNR), and  $\mathbf{H}$  is the  $M \times N$  matrix whose elements  $\{h_{nm}\}$  represent the channel gains between pairs of transmit and receive antennas. The achievable data rate depends on the rank of  $\mathbf{H}$  [6].

For large SNR and large  $N$  and  $M$ , the capacity tends to the value  $r \log_2 \rho$ , where  $r = \text{rank}(\mathbf{H})$ . When the elements of  $\mathbf{H}$  are independent and identically distributed, the rank  $r = N$ . Hence, in this ideal scenario of independent fading, the data rate grows linearly with the number of transmit antennas [3]. Ideally, the  $M$  receive antennas can provide  $M^{\text{th}}$ -order diversity reception for each of the  $N$  transmitted signals in addition to whatever implicit diversity the channel has to offer [5]. Since there is no orthogonal structure imposed on the signals by the transmitter, the received signals contain inter channel interference. The receiver must therefore be able to separate the  $N$  signals and at the same time take advantage of the inherent signal diversity. The rule of thumb is that in order to ensure independent fading, the antennas have to be separated by at least half a wavelength at the receiver and as much as several wavelengths at an elevated transmitting base station [7].

In this context, we will discuss the performance of three detectors namely ML, ZF-SIC and MMSE-SIC detectors. We would focus our discussion to the experimental results carried out to MIMO systems and then try to analyze which of the detectors have a better performance in terms of BER for a given SNR. The article is organized as follows. In section II describes the details of MIMO system model. Section III describes MIMO non-linear detectors. Section IV, we discuss the results and performance comparisons of different receivers and section V, we conclude our discussion.

## 2 MIMO SYSTEM MODEL

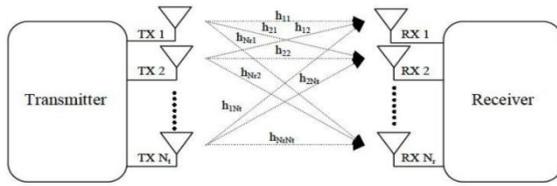


Fig. 1 MIMO system for  $n_T$  transmit antenna and  $m_R$  receive antenna

Above figure shows that MIMO system consists of  $N$  transmitting antennas and  $M$  receiving antennas. For the Channel Rayleigh flat fading channel is considered.

Let us consider for  $2 \times 2$  MIMO System

The received signal on the first receive antenna is

$$y_1 = h_{11}x_1 + h_{12}x_2 + n_1 \quad (2)$$

The received signal on the second receive antenna is

$$y_2 = h_{21}x_1 + h_{22}x_2 + n_2 \quad (3)$$

Where  $y_1$  and  $y_2$  are the received symbol on the first and second antenna respectively,  $h_{11}$  is the channel from  $1^{st}$  transmit antenna to  $1^{st}$  receive antenna,  $h_{12}$  is the channel from  $2^{nd}$  transmit antenna to  $1^{st}$  receive antenna,  $h_{21}$  is the channel from  $1^{st}$  transmit antenna to  $2^{nd}$  receive antenna,  $h_{22}$  is the channel from  $2^{nd}$  transmit antenna to  $2^{nd}$  receive antenna,  $x_1$  and  $x_2$  are the transmitted symbols and  $n_1$  and  $n_2$  is the noise on  $1^{st}$  and  $2^{nd}$  receive antennas respectively.

Eqn (2) and Eqn (3) can be represented in matrix form

$$[y_1 \ y_2] = [h_{11} \ h_{12} \ h_{21} \ h_{22}] [x_1 \ x_2] + [n_1 \ n_2]$$

Therefore, the received vector can be expressed as

$$y = Hx + n \quad (4)$$

### 3. MIMO EQUALIZATION TECHNIQUES

The MIMO channel assumed flat fading – simple channel with only one tap. so, the convolution operation reduces to a simple multiplication. For the  $i^{th}$  transmit antenna to  $j^{th}$  receive antenna, each transmitted symbol gets multiplied by a randomly varying complex number  $h_{j,i}$ . As the channel under consideration is a Rayleigh channel, the real and imaginary parts of  $h_{j,i}$  are Gaussian distributed having mean  $\mu_{h_{j,i}} = 0$  and variance  $\sigma_{h_{j,i}}^2 = \frac{1}{2}$  on the receive antenna, the noise  $n$  has the Gaussian probability density function with,

$$p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-\mu)^2}{2\sigma^2}}$$

$$\text{with } \mu = 0 \text{ and } \sigma^2 = \frac{n_0}{2}$$

The channel  $h_{j,i}$  is known at the receiver.

#### A. Maximum Likelihood (ML) Equalizer

The Maximum Likelihood receiver tries to find  $x$  which minimizes,

$$J = |y - Hx|^2$$

$$J = |[y_1 \ y_2] - [h_{11} \ h_{12} \ h_{21} \ h_{22}] [x_1 \ x_2]|^2$$

Since the modulation is BPSK, the possible values of  $x_1$  is  $+1$  or  $-1$ . Similarly  $x_2$  also take values  $+1$  or  $-1$ . So, to find the Maximum Likelihood solution, we need to find the minimum from the all four combinations of  $x_1$  and  $x_2$ .

$$J_{+1,+1} = |[y_1 \ y_2] - [h_{11} \ h_{12} \ h_{21} \ h_{22}] [+1 \ +1]|^2$$

$$J_{+1,-1} = |[y_1 \ y_2] - [h_{11} \ h_{12} \ h_{21} \ h_{22}] [+1 \ -1]|^2$$

$$J_{-1,+1} = |[y_1 \ y_2] - [h_{11} \ h_{12} \ h_{21} \ h_{22}] [-1 \ +1]|^2$$

$$J_{-1,-1} = |[y_1 \ y_2] - [h_{11} \ h_{12} \ h_{21} \ h_{22}] [-1 \ -1]|^2$$

The estimate of the transmit symbol is chosen based on the minimum value from the above four values i.e. if the minimum is,  $J_{+1,+1} = [1 \ 1]$ , if the minimum is,  $J_{+1,-1} = [1 \ 0]$ , if the minimum is,  $J_{-1,+1} = [0 \ 1]$ , if the minimum is,  $J_{-1,-1} = [0 \ 0]$ .

#### B. Zero Forcing Successive Interference Cancellation (ZF-SIC) Equalizer

To solve for  $x$ , we know that we need to find a matrix  $W$  which satisfies  $WH=I$ . The Zero Forcing (ZF) linear detector for meeting this constraint is given by,

$$W = (H^H H)^{-1} H^H$$

For Zero Forcing with Successive Interference Cancellation, the receiver can obtain an estimate of the two transmitted symbols  $x_1$  and  $x_2$ , i.e.,

$$[x_1 \ x_2] = (H^H H)^{-1} H^H [y_1 \ y_2]$$

Take one of the estimated symbols (for example  $\hat{x}_2$ ) and subtract its effect from the received vector  $y_1$  and  $y_2$ , i.e.

$$\begin{aligned} [r_1 \ r_2] &= [y_1 - h_{12}x_2 \ y_2 - h_{22}x_2] \\ &= [h_{11}x_1 + n_1 \ h_{21}x_1 + n_2] \end{aligned}$$

Expressing in matrix notation,

$$[r_1 \ r_2] = [h_{11} \ h_{21}] x_1 + [n_1 \ n_2]$$

$$r = Hx_1 + n$$

The equalized symbol is,

$$\hat{x}_1 = \frac{H^H r}{H^H H}$$

### C. Minimum Mean Square Error Successive Interference Cancellation (MMSE-SIC) Equalizer

Similar to the ZF detector, the Minimum Mean Square Error (MMSE) approach tries to find a coefficient  $W$  for the equation of  $y$  which minimizes the criterion,

$$E\{[Wy - x][Wy - x]^H\}$$

Solving,

$$W = [H^H H + N_0 I]^{-1} H^H$$

Using the Minimum Mean Square Error (MMSE) equalization, the receiver can obtain an estimate of the two transmitted symbols  $x_1, x_2$ , i.e.

$$[x_1 \ x_2] = [H^H H + N_0 I]^{-1} H^H [y_1 \ y_2]$$

In MMSE Successive Interference Cancellation, the receiver arbitrarily takes one of the estimated symbols (for example the symbol transmitted in the second spatial dimension,  $x_2$ ), and subtract its effect from the received symbol  $y_1$  and  $y_2$ . Once the effect of  $\hat{x}_2$  is removed, the new channel becomes a one transmit antenna, 2 receive antenna case and can be optimally equalized by Maximal Ratio Combining (MRC).

#### 4. SIMULATION PARAMETER AND RESULTS

In this section analyzed BER performance for BPSK modulation technique in MIMO ML, ZF-SIC and MMSE-SIC receiver. For analytic analysis taken parameters are as below:

Table 1(Simulation Parameters)

NTx	2
NRx	2
Symbols	1000000
Noise	Gaussian noise
Channel	Rayleigh flat fading channel
SNR	0 to 25
Modulation	BPSK

### Results

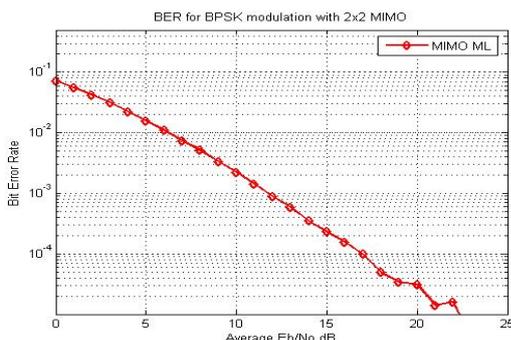


Fig. 2 BER vs SNR for 2x2 MIMO ML (BPSK modulation in Rayleigh channel)

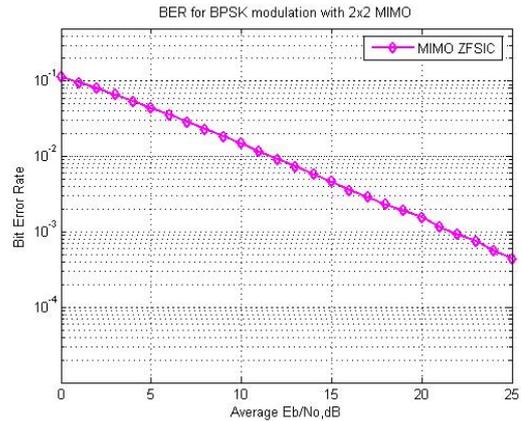


Fig. 3 BER vs SNR for 2x2 MIMO ZF-SIC (BPSK modulation in Rayleigh channel)

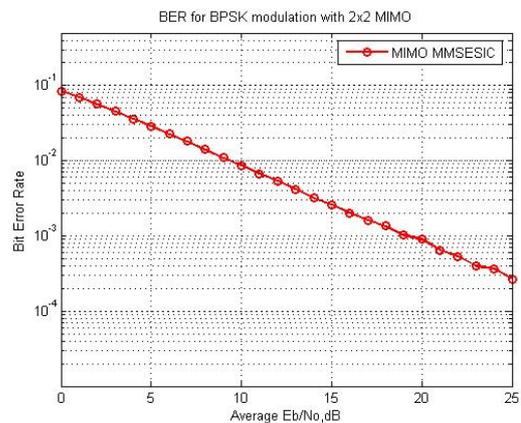


Fig. 4 BER vs SNR for 2x2 MIMO MMSE-SIC (BPSK modulation in Rayleigh channel)

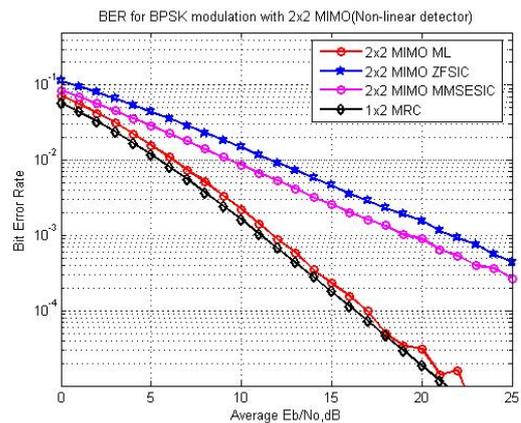


Fig. 5 BER comparison of 2x2 MIMO Nonlinear detectors (BPSK modulation in Rayleigh channel)

### 5. CONCLUSION

The BER Performance of the 2x2 MIMO systems in Rayleigh Wireless channel under BPSK modulation scheme is gradually changed by changing the detector. Analysis for different BER for ML, ZF\_SIC and MMSE-SIC are compared and also analyzed for the MRC. At BER~10<sup>-3</sup> the performance of MIMO system is integrated

with ZF-SIC and MMSE-SIC equalizers, MMSE-SIC performs better (Fig.3 & Fig.4) but by adopting ML Equalizer for the MIMO technique, the system achieved best performance (Fig.3 & Fig.5). The study confirms that the better BER performance is achieved if receiver diversity is more than transmission diversity under the MIMO conditions. So MRC gives best results as compared to others (Fig 5). Also the complexity of ML decoder goes on increasing as move to higher modulation schemes. Values of SNR at  $BER \sim 10^{-3}$  are shown in below table.

BER	ML	MMSE-SIC	ZF-SIC
$10^{-3}$	12 dB SNR	19 dB SNR	21 dB SNR

At BER  $10^{-3}$  performance of detectors analyzed that

ML > MMSE-SIC > ZF-SIC.

## 6. REFERENCES

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