Radiation and Mass Transfer Effects on MHD Boundary Layer Flow due to an Exponentially Stretching Sheet with Heat Source – A Keller Box Approach

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Abstract - A numerical study on boundary layer flow behavior, heat and mass transfer effects on MHD boundary layer flow of a viscous incompressible and radiating fluid over an exponentially stretching sheet with heat source is presented in this paper. The initial governing boundary layer equations are transformed to a system of ordinary differential equations, which are then solved numerically by applying the implicit finite difference scheme known as Keller Box method. A parametric study is conducted and so that Numerical results are obtained for the velocity, temperature and concentration as well as the skin-friction coefficient, the local Nusselt number and local Sherwood number for different values of the material parameters, namely, the magnetic parameter, heat source parameter, radiation parameter, Schmidt number and Prandtl number are discussed in detail. The results are presented in both graphical and tabular forms.

Keywords - Exponentially Stretching Sheet, MHD, Heat and Mass Transfer, Heat Source, Radiation. Keller Box method.

1. INTRODUCTION

The industrial processes like hot rolling, wire drawing, spinning of filaments, metal extrusion, crystal growing, glass fiber production, cooling of a large metallic plate in a bath, which may be an electrolyte, etc. to require the study of flow and heat transfer over a stretching surface. In all these cases, the quality of final product depends on the surface heat transfer rate and the skin friction coefficient. Magyari and Keller [1] investigated the steady boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution. Crane [2] was the first to consider the boundary layer flow caused by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point. The heat transfer aspect of this problem was investigated by Carragher and Crane [3], under the conditions when the temperature difference between the surface and the ambient fluid is proportional to a power of the distance from a fixed point. Shanker and Kishan [9] presented the effect of mass transfer on the MHD flow past an impulsively started infinite vertical plate. Bhaskara Reddy and Bathaiah [10, 11] analyze the Magnetohydrodynamic free convection laminar flow of an incompressible Viscoelastic fluid. Later, he was studied the MHD combined free and forced convection flow through two parallel porous walls. Elabashbeshy [12] studied heat and mass transfer along a vertical plate in the presence of magnetic field. Gangadhar and Bhaskar Reddy [13] analyzed the problem of chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate in a porous medium with suction. The heat source/sink effects in thermal convection, are significant where there may exist a high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reactions. Recently, Poornima and Bhaskar Reddy [27] presented an analysis of the radiation effects on MHD free convective boundary layer flow of nanofluids over a nonlinear stretching sheet. However, the interaction of radiation with mass transfer due to a stretching sheet has received little attention.

Renuka Devi and Bhaskar Reddy[31] presented analysis of the radiation and mass transfer effects on MHD boundary layer flow due to an exponentially stretching sheet with heat source. Hence, the aim of the present study is to analyze the effect of thermal radiation and mass transfer on the steady magneto hydrodynamic (MHD) boundary layer flow due to an exponentially stretching sheet in the presence of heat source or sink. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables and the resultant problem is solved numerically by applying the implicit finite difference scheme known as Keller Box method. A parametric study is conducted and so that Numerical results are obtained for the velocity, temperature and concentration as well as the skin-friction coefficient, the local Nusselt number and local Sherwood number for different values of the material parameters, namely, the magnetic parameter, heat source parameter, radiation parameter, Schmidt number and Prandtl number, are discussed in detail.

2. FORMULATION OF THE PROBLEM

The present problem is based on a steady two-dimensional flow of an incompressible viscous, electrically conducting and radiating fluid past an exponentially stretching sheet is considered. The sheet is placed in a quiescent ambient fluid of uniform surface temperature and concentration T_∞ and C_∞.

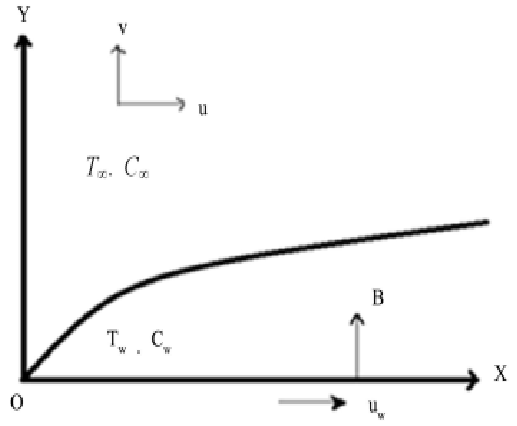

Figure 1. Physical modal and coordinate system.

The x-axis is taken along the plate and y-axis normal to it. A variable magnetic field of strength B(x) is applied transversely and the induced magnetic field is assumed to be neglected, which is justified for MHD flow with small magnetic Reynolds number. Hall effects and Joule heating are also negligible. The level of concentration of foreign mass is assumed to be low, so that the Soret and Dufour effects are negligible. Under these assumptions along with the Boussinesq and boundary layer approximations, the system of equations, which models the flow is given byand radiating fluid past an exponentially stretching sheet is considered. The sheet is placed in a quiescent ambient fluid of uniform surface temperature and concentration and . The x-axis is taken along the plate and y-axis normal to it. A variable magnetic field of strength B(x) is applied transversely and the induced magnetic field is assumed to be neglected, which is justified for MHD flow with small magnetic Reynolds number. Hall effects and Joule heating are also negligible. The level of concentration of foreign mass is assumed to be low, so that the Soret and Dufour effects are negligible. Under these assumptions along with the Boussinesq and boundary layer approximations, the system of equations, which models the flow is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \mathcal{V}\,\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\,B(x)}{\rho}\,u \tag{2}$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\,\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\,\frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho c_p}\,(T - T_\infty) \tag{3}$$

$$u\,\frac{\partial C}{\partial x} + v\,\frac{\partial C}{\partial y} = D\,\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The boundary conditions for the velocity, temperature and concentration fields are

$$u = U_w = U_0 e^{\frac{x}{L}},\ v = 0,\ T = T_w = T_\infty + T_0 e^{\frac{x}{2L}}$$

$$C = C_w = C_\infty + C_0 e^{\frac{x}{2L}} \qquad \text{at } y = 0$$

$$u \to 0,\ T \to T_\infty,\ C \to C_\infty \text{ as } y \to \infty \tag{5}$$

where u and v are the velocity components along the x and y axes, respectively, T - the temperature of the fluid and C - the fluid concentration in the boundary layer, v - the kinematic viscosity, ρ - the fluid density, Cp - the specific heat, $B(x)$ - the magnetic field of constant strength, qr - the radiative heat flux, L - the reference length,$U0$ - the reference velocity,$T0$ - the reference temperature,$C0$ - the reference concentration, Tw - the temperature uniform of the sheet, Cw - the concentration uniform of the sheet, D - the coefficient of mass diffusivity and k - the thermal conductivity of the fluid. By using the Rosseland approximation (Brewster [28]), the radiative heat flux qr is given by

$$q_r = -\frac{4\sigma^*}{3K'} - \frac{\partial T^4}{\partial y} \tag{6}$$

where σ^* is the Stefan-Boltzmann constant and K' - the mean absorption coefficient. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficiently small, then the equation (7) can be linearized by expanding T^4 into the Taylor series about T_∞, which after neglecting higher order terms takes the form

$$T^4 \cong 4T_\infty^3\,T - 3T_\infty^4 \tag{7}$$

In view of the equations (7) and (8), the equation (3) reduces to

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \left(\frac{k}{\rho c_p} + \frac{16\sigma^* T_\infty^3}{3\rho c_p\,K'}\right)\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p}\,(T - T_\infty) \tag{8}$$

To obtain similarity solutions. It is assumed that the magnetic field B(x) is of the form

$$B(x) = B_0 e^{\frac{x}{2L}}$$

where B0 is the constant magnetic field.

The continuity equation (1) is satisfied by the Cauchy Riemann equations

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{9}$$

where ψ(x,y) is the stream function.

In order to transform the equations (2), (4) and (8) into a set of ordinary differential equations, the following similarity transformations and dimensionless variables are introduced (Sajid and Hayat [25]).

$$\psi\,(x,y) = \sqrt{2vU_0L}\; e^{\frac{x}{2L}}\, f(\eta), \quad \eta = \left(\frac{U_0}{2vL}\right)^{\frac{1}{2}} e^{\frac{x}{2L}}\, y$$

$$u = U_0 e^{\frac{x}{L}}\, f'(\eta),\; v = -\left(\frac{vU_0}{2L}\right)^{\frac{1}{2}} e^{\frac{x}{2L}}\,(f(\eta) + \eta f'(\eta))$$

$$T = T_\infty + T_0 e^{\frac{x}{2L}}\,\theta(\eta),\; Q = \frac{Q_0}{\rho c_p},\; C = C_\infty + C_0 e^{\frac{x}{2L}}\,\phi(\eta)$$

$$M = \frac{2\sigma B_0^2 L}{\rho U_0},\; P_r = \frac{\rho V c_p}{k},\; S_c = \frac{v}{D},\; R = \frac{4\sigma^* T_\infty^3}{K\,K'} \tag{10}$$

where $f\,(\eta)$ is the dimensionless stream function, θ - the dimensionless temperature, ϕ - the dimensionless concentration, η - the similarity variable, *M*- the magnetic parameter, *Pr* - the Prandtl number, *Q* - the heat source or sink parameter, *Sc*- the Schmidt number and *R*- the radiation parameter. In view of the equations (9) and (10), the equations (2), (4) and (8) transform into

$$f''' + ff'' - 2f'^2 + Mf' = 0 \tag{11}$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + Prf\theta' - Prf'\,\theta + PrQ\,\theta = 0 \tag{12}$$

$$\phi'' + Scf\phi' - Scf'\phi = 0 \tag{13}$$

The transformed boundary conditions can be written as

$$f = 0,\, f' = 1,\, \theta = 1,\, \phi = 1 \text{ at } \eta = 0$$

$$f' = \theta = \phi = 0 \text{ at } \eta \to \infty \tag{14}$$

The main physical quantities of interest are the skin friction coefficient $f''(0)$, the local Nusselt number $-\theta'(0)$and the Sherwood number $-\phi'(0)$ which represent the wall shear stress, the heat transfer rate and mass transfer rate at the surface, respectively. Our task is to investigate how the values of $f''(0), -\theta'(0)$ and $-\phi'(0)$ vary with the radiation parameter *R*, magnetic parameter *M* and Prandtl number *Pr.*

3. NUMERICAL SOLUTION

The set of coupled non-linear governing boundary layer equations (11) - (13) together with the boundary conditions (14) are solved numerically by using implicit finite difference scheme known as Keller Box method. First of all, higher order non-linear differential Equations (11) - (13) are converted into simultaneous linear differential equations of first order and they are further, transformed into initial value problem. The resultant initial value problem is solved by employing implicit finite difference scheme known as Keller Box method. The step size $\Delta\eta$=0.04 is used to obtain the numerical solution with four decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and the Sherwood number, which are respectively proportional to $f''(0), -\theta'(0)$ and $-\phi'(0)$, are also sorted out and their numerical values are presented in a tabular form.

4. RESULTS AND DISCUSSION

In order to get a clear insight of the physical problem, the velocity, temperature and concentration have been discussed for various numerical values which encountered in the governing equations. The effects of various parameters on the velocity are depicted in Fig. 2.From the graph it is observed that for fixed values of Pr, R, Q ,Sc and for increasing magnetic parameter M=0,1,2,4 the velocity profiles decreases and the curve meets x-axis at affixed value. From Fig .3 to 6 depicts effects of various parameter on temperature . The effects of various parameters on the concentration are depicted in Figs. 7-8. In Fig 3 we observe increase in magnetic parameter M=0, 1,2 4 with fixed values of Pr , R ,Q, Sc ,the temperature profile increases initially and then decreases and meets x-axis at finite distance. From Fig .4.we observe for fixed values of M,R,Q and Sc and increasing values of Pr=1,2,3,4,the thermal boundary layer thickness decreases and meet x-axis at a finite value.Fig.5. depicts the effect of radiation parameter R on the temperature as R takes the values 0,1,2,3 with fixed M=2,Pr=1,Q=0.5 and Sc=0.22 the temperature profile decreases and meets x-axis .It is observed from figure 6,increase in heat source parameter Q with M=2,Pr=R=1,Sc=0.22 leads to increase in temperature profile initially and then starts decreasing and meets

x-axis. It is noticed that as the heat source parameter increases, the temperature increases. The effect of the magnetic parameter (*M*) on the concentration field is illustrated in Fig.7. depicts the effect of concentration parameter C with

Pr=R=1,Q=0.5,Sc=0.22 as the magnetic parameter M=0,1,2,4 increases the concentration is found to be increasing and leads to increase in concentration profile initially and then starts decreasing and meets x-axis. The effect of the Schmidt number *(Sc)* on the concentration field is illustrated in Fig. 8. depicts the effect of concentration parameter C with Pr=R=1,Q=0.5,M=2 as the

Schmidt number Sc =0.24,0.62,0.78,2.62 increases concentration boundary layer thickness decreases and meets x-axisThe present results presented in Table 1., and are compared with those of Magyari and kellar [1], Bhaskar Reddy [31] and Bidin and Zazar [26] and they agree closely with negligible amount of error.

5.CONCLUSION

The effect of radiation and mass transfer on MHD boundary layer flow due to an exponentially stretching sheet with heat source/sink are investigated. The numerical approximations are obtained by using Keller box method .They agree closely with the results which are available in the literature and presented in Table[1].

The following conclusions are drawn.

- With increasing magnetic field intensity the momentum boundary layer thickness decreases while both thermal and concentration boundary layer thickness increases.
- The effect of radiation reduces temperature.
- The concentration reduces as Schmidt number increases.
- The magnetic field, radiation and Prandtl number enhances the heat transfer rate.

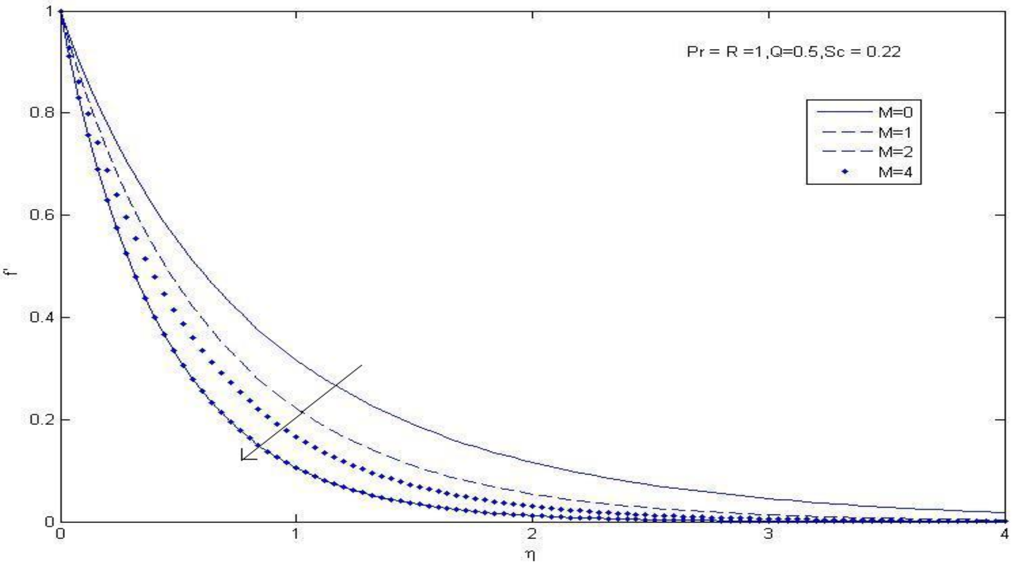

Fig.2 Velocity for different values of M

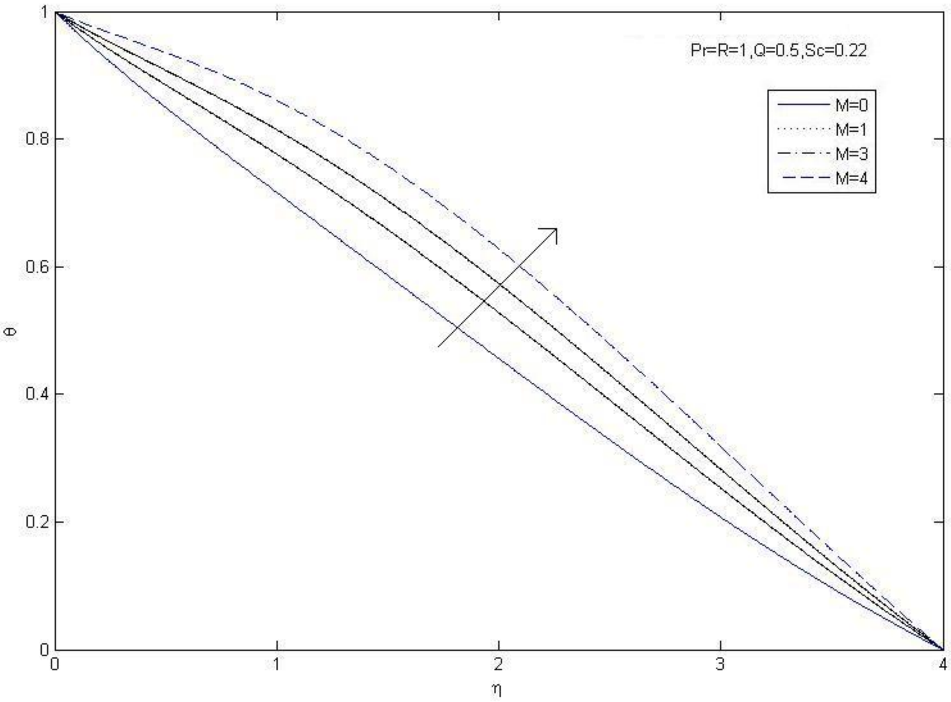

Fig.3Temperature for different values of *M*

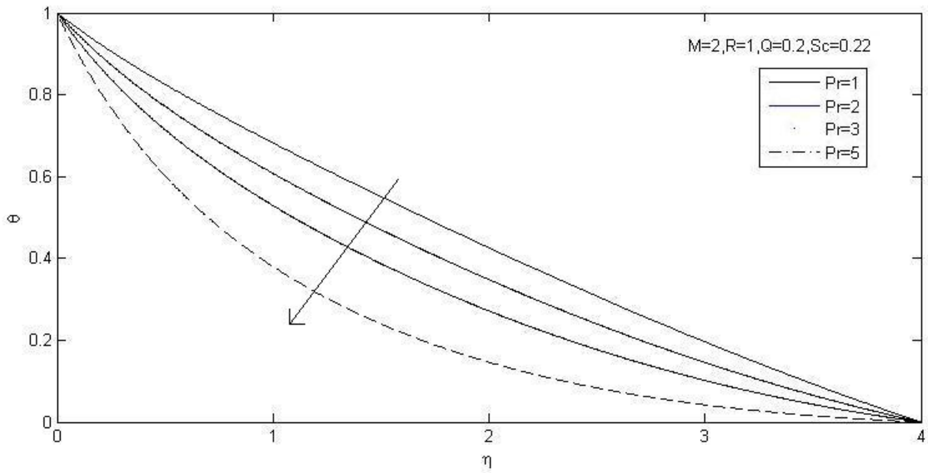

Fig.4 Velocity for different values of *Pr*

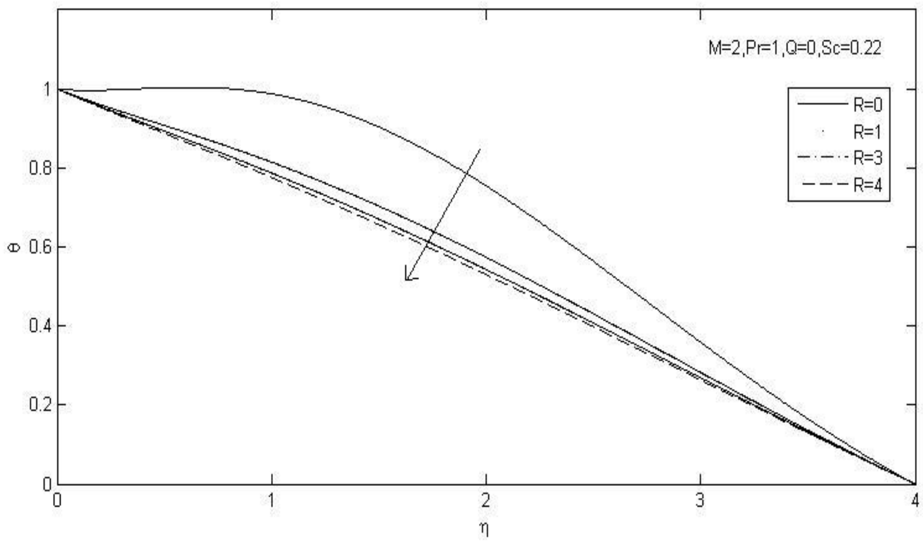

Fig.5 Temperature for different values of *R*

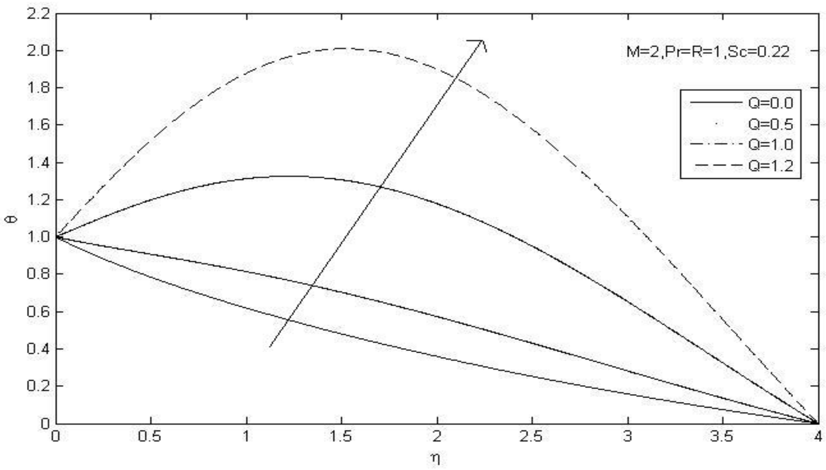

Fig.6 Temperature for different values of *Q*

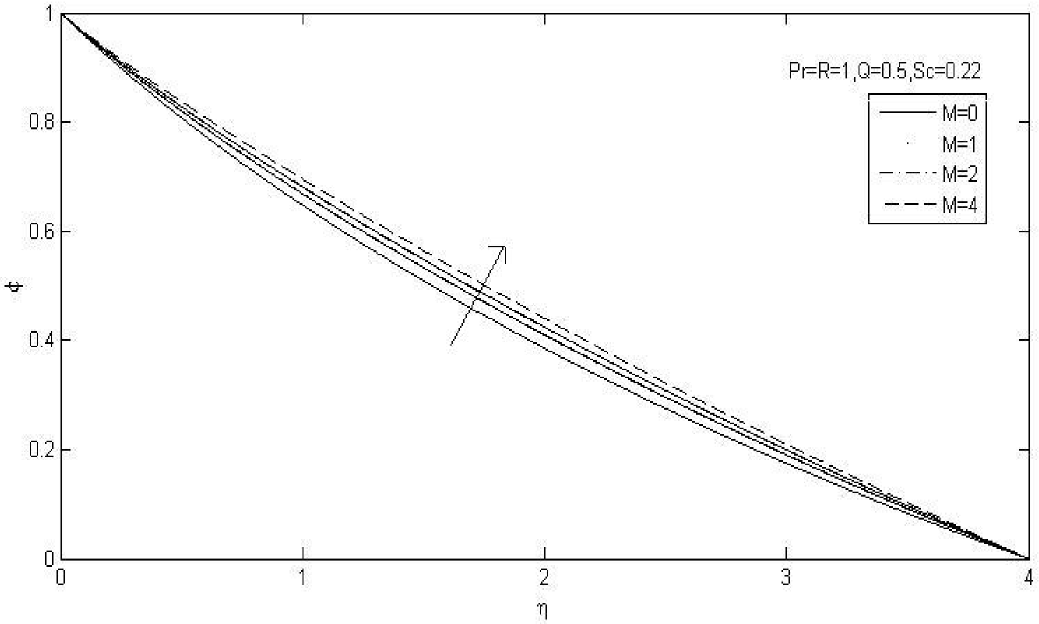

Fig.7 Concentration for different values of *M*

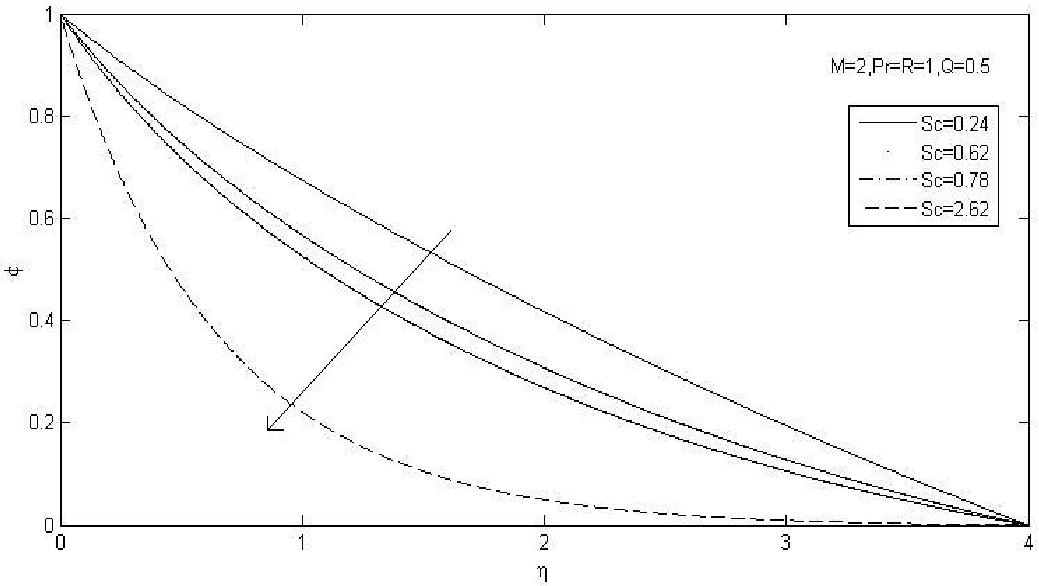

Fig.8 Concentration for different values of *Sc*

Table 1 Numerical values of $\theta'(0)$ at the sheet for different values of R,M and Pr when Q =0 and Sc =0, Comparison of the present results with that of Magyari and Kellar [1],Bhaskar Reddy[31] and Bidin and Nazar [26]

R	M	Pr	Present Results	Shooting Techniques[1]	Runge-Kutta Method[31]	Keller Box Method[26]

0	0		-0..9548		-0.957433	
		1	-1.4715		-1.47898	-0.9548
		2	-1.8692	-0.954782	-1.86443	-1.4714
		3	-2.5005		-2.500615	-1.8691
		5	-3.6619	-1.869075	-3.60024	
	1	10	-0.8634	-2.500135	-0.861390	
	0	1	-0.5311	-3.660379	-0.563196	
1	1		-0.4524		-0.450533	-0.5315

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