A Brief Discussion on Equality In Multi Scale Iterative Morphology & Structuring Element De-Composition

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Abstract: This paper discusses about primitive morphological operations in multi scale environment. The morphological operations are main source for defining composite morphological operations. These are applied in various image processing operations. In addition to them morphological operations are having special applications also. So the study of morphological operations in various environments will provide broad look of these operations, which throws light on understanding of these basic principles which will help in further new applications of this mathematical morphology. So in this paper morphological operations are discussed in a new dimension.

KEYWORDS: erosion ,dilation, open, close, multi scale, mathematical morphology, structuring element.

I. INTRODUCTION

If we observe carefully, the human beings have the desire of recording incidents, through images. Their view may be for the purpose of future generation. Images also, played the role of symbols of languages, for communication purpose.

The early cavemen documented some of the incidents through images in the caves. They documented some of the incidents of their routine life, on stones, by using primitive tools. Important incidents such as battles, routine incidents such as food habits were recorded by them, on stones. These provide record, which is historically very important, of early human civilization. The images drawn by primitive tools by Egyptians, Indians, have provided a lot of valuable information, for historians, about civilizations.

After this, paints or inks were invented. The human beings started to record scenes, incidents through these paints and inks. Letter on J. B. Porta, an Italian Philosopher, during the II half of 18th century, by mean of an accidental discovery, was able to assemble a camera like equipment by mirrors and lens, which is the first step towards the modern day photography. At the same time a France scientist observed silver chloride characteristics with respect to light. After two centuries Alexander Charles extended above concept, and produced simple photo graphs.

After one century, at around 1835 Henry Fox Talbot extended above concepts, using silver nitrate, extended the design of camera, and modern photography was born from this experiment, which is presented in royal society.

This technology is used to record incidents of U.S. civil war, or, to record incidents of wealthy people, but not reached to a common man, due to complex chemical process, for the development of photographs till “KODAK” has entered in 1884.. Later on research is done on motion pictures by Thomas A. Edison & William Kennedy Laurie Dickson, which is foundation for modern movie technology. Actually the first step for images processing was laid during Second World War. Technical experts, who are trained specially, are used to improve quality of image. They are specially trained in object recognition, they used to identity targets, manually. So, it is first step in image processing. After invention of digital computer, digital image processing came into existence. NASA, in early 1960’s, got images from Space Crafts, Ranger 7, of the Lunar Surface, in thousands. These images were processed to minimize distortions. This is initial digital I.P. work, using a computer. This work was done in NASA’s JET propulsion laboratory (JPL), in California.

This initial digital images processing work was very satisfactory. So, NASA continued it’s funding, resulting in the development of digital image processing area.

The reduction in Hardware cost, mass production of chips, reduction in memory cost, reduction in size of computers , boosted the development of Digital Image Processing area.

So, researches in general have been showing interest and developed algorithms for image smoothening, edge enhancement, image compression, image segmentation, 2D to 3D conversion etc., Now a day, it is having applications from entertainment area to medical area.the detailed explanation is given in author's papers.

At the same time mathematical morphology emerged and developed separately, with some other interests and motivations. The purpose of this area is different. But later on, it is identified that the mathematical morphology is having very important applications in image processing. So, mathematical morphology is considered now, a very important branch of image processing.

Actually J. SERRA (1) and MATHERON (2) are founders of mathematical morphology. They have explained all the fundamentals of mathematical morphology in their books.

Actually the primitive operations are EROSION & DILATION. The composite operations are open and close. All these are explained in chapters 1 and 2. There are some more composite operations, like thinning, skeletenization etc. But the work is limited to erosion, dilation, open, close.

Mr. H.J.A.M. HEIJMANS has given a detailed discussion of these operations in 4. Till now the light is thrown on the fundamentals of mathematical morphology (1,……. 4).

The morphological operations are suitable to apply on binary images only. Actually, applications of morphological operations were extended by SERRA also. Later STERNBERG concentrated in this area. In depth study was done (the theoretical analysis) by J.A.M HEIGMANS in this area. PETROS MARAGOS has discussed about morphology also. PETROS MARAGOS has discussed about morphology and given theoretical analysis.

For elimination or minimization of noise in the images a lot of research is done. The researchers developed algorithms for smoothening with detail preservation and for edge enhancement also. some resesrchers developed morphological algorithms for elimination of salt and pepper noise ,and impulse noise also. It has entered into medical area also

the detailed references are available in the other papers of author.(6 to 18).

The mathematical morphology has entered in to some more areas like soft morphology,fuzzy morphology,flat morphology etc. some of the work done by the author in soft morphology is referred in references.

II. DEFINITIONS

The primitive morphological operations are dilation and erosion. By means of these operations only, all the remaining morphological operations may be defined. These two morphological operations play the role of bricks, for a house.

2.1. Dilation: - These operations may be defined in so many ways. Different researchers defined this operation in different ways.

2.1.1. <u>Def. 1</u>:- Let A and B be subjects of E^N (where N is Space) the dilation of A by B, is denoted by $A \oplus B$ and is defined by $A \oplus B = \{C / C = a + b$ for some $a \in A$ and $b \in B\}$

<u>Def 2</u>:- $A \oplus B = U\ (A)_b$

$b \in B$ Where A is the image and B is the structuring element.

Here $(A)_b$ means, translation of A by b, defined as

$(A)_b = \{C / C = a + b;\ a \in A\}$

<u>Def 3</u>:- $(I \oplus S)\,[x, y] = 1$ if $|\, I \cap S'_{(x, y)} \,| \geq |$

= 0 otherwise.

Here, I is the image

S: structuring element

S' : reflection of S about the origin

[If S.E. is having origin, at its centre point then S= S'.]

I (x, y) denotes image pixel value at the coordinate (x, y)

|Z| denotes the cardinality of the set Z;

$S_{(x, y)}$: S translated by the displacement {x,y}.

2.2. Erosion: This morphological operation also defined in so many ways, by different researchers.

2.2.1 Def 1):- The erosion of A by B is denoted by $A \ominus B$, and is defined by

$A \ominus B = \{x / x + b \in A$ for every $b \in B\}$ Here $x \in E^N$ when E^N= N space.

Def 2):- $A \ominus B = \{x /$ for every $b \in B$, there exists and $a \in A$, such that $x = a - b\}$

Def 3):- $A \ominus B = \{x / (B)_x \subseteq A\}$. Here A is image, B is S.E.

Here $x \in E^N$ $(B)_x$: Translation of B by "x"

Def4):- $A \ominus B = \bigcap_{b \in B} (A)_b$ Here "A" is the image and B is the S.E.

$(A)_b$: Translation of A by b

Def 5):- $(I \ominus S)[x, y] = 1$ If $|I \cap S_{(x, y)}| = |S|$

= 0 other wise

Here I is image and S is S.E. I(x, y) denotes image value at coordinate (x, y) |Z| denotes the cardinality, of the set Z. $S_{(x,y)}$: S translated by the displacement (x, y)

III. MULTI SCALE ENVIRONMENT

DISCUSSION ON MULTI SCALE SOFT MORPHOLOGY

In the process of understanding the objective world, the appearance of an object does not depend only on the object itself, but also on the scale that the observer used. It seems that appearance under a specific scale does not give sufficient information about the essence of the percept, we want to understand. If we use a different scale, to examine this percept, it will usually have a different appearance. So, this series of images and its changing pattern over scales reflect the nature of the percept.

The S.E. dimension can be anything. It depends upon situation, requirement, and context etc. It can be $1/1, 2/2, 3/3, 4/4, 5/5, 6/6, 7/7, \ldots\ldots\ldots$

In some situations, particularly square grid is chosen, it can be $3/3, 5/5, 7/7, 9/9, 11/11, 13/13, \ldots\ldots\ldots$

The S.E.'s, having series, and in increasing size [like mentioned above] is called multi scale S.E.'s and the morphological approach (operations) dealing with multi scale S.E.'s is called multi scale morphology. As the size of the S.E. is more, its impact upon image will be more. For example, amount of expansion by applying dilation operation is more on an image, if we apply $5/5$ S.E., compared to amount of expansion of image, by dilating by $3/3$ S.E.

REVIEW ON MULTI SCALE SOFT MORPHOLOGY

Till now, some amount of research is done in this area, and it is applied in so many areas. In mathematical morphology also, a new area multi scale mathematical morphology is developed, and applied in so many areas like smoothening, edge enhancement, analysis of radar imagery, remote sensing, medical image processing etc.

PETROS MARAGOS entered into multi scale morphology, in addition to other areas. He explained about changes of shapes, as the scale is changed. He explained the applications of MSMM, and back ground mathematics. He explained about application of MSMM in skeletenization also. He extended these concepts to gray scale also.

MING – HUA CHEN & PING – GAN YAN explained Erosion, Dilation, Open, Close in multi scale environment, with diagrams (results), mathematical analysis, as well as symbolic conventions.

PAUL. T. JACKWAY etc. provided one type of analysis in MSMM. They discussed how to relate the results of one scale with the results at different scale. They have provided this analysis with good examples, using Erosion/Dilation morphological operations. KUN WANG etc. proposed an algorithm, for edge detection in the presence of Gaussian noise & salt – pepper noise in multi scale morphological environment. The experimental results are better than that of conventional algorithms. The same authors KUNWANG etc. proposed another algorithm for edge detection which will function better in Gaussian, salt - paper noise environment, in MS morphological approach.

KIM WANG and others discussed an edge detection algorithm, in multi scale environment, which is suitable to apply on brain MRI, in noisy environment.

ZENG PINGPING etc. proposed another algorithm, for edge enhancement in multi scale morphological approach, using order morphology also, which is suitable to apply in noisy environment also. ZHEANHUA LI; & others discussed another technique for edge enhancement, in MS morphological environment.

PANCHAO WU & others proposed another algorithm, for edge detection in noisy environment using MS MM & WAVELET transforms.

GAO LI etc proposed an adaptive algorithm for edge detection of a color image (In HIS space) in MSMM environment. CHEN JIN LONG, etc. proposed another methodology for edge detection in multi structure and multi scale mathematical morphology environment.

HAI LONG HUANG etc. proposed an algorithm for suppression of noise and preserve edges using multi share and multi scale mathematical morphology environment. HAI LONG HUANG etc. proposed an algorithm for suppression of noise and preserve edges using multi share and multi scale structure elements using different directions and sizes of S.E.'s.

These MSMM techniques are extended to segmentation also. DEBAY LE, J. etc extended MSMM for segmentation using adaptive technique and MARC DROSKE etc. also used MSMM for segmentation. H UANG, R. etc. discussed extension of MSMM to 3D. They discussed and designed algorithm for volume segmentation. For this purpose, they have designed spherical S. E.'s at various sizes. LETITIA, S; etc. applied MSMM for road segmentation from satellite aerial images.

MSMM is having, application in medical area also. DA WEI QI etc. shown an application in medical I.P. for edge detection in noisy environment, which gives better results, compared to traditional pictures. FEI ZHANG etc., given another algorithm, suitable for ECG analysis, in impulse noise environment using MSMM. DAWEI QI proposed another algorithm, for medical analysis environment. ZA BI HI, S.M etc. (148) discussed application of MSMM for retinal vessel segmentation.. DAWEI QI etc HAI YAN GU; etc WEIPING HOU etc discussed the applications of MSMM in wood analysis they have done wood decay estimations, defect identification of wood, etc. RUJIANG HAO etc. used MSMM open operation for identification of defects of the rolling beatings. YING ZHANG etc. used MSMM to do analysis of results of turbine rotor experiment. In noise environment also, it provides good results [strong edges].The detailed references and explanations are available in my other papers, which are given in reference.

IV. STRUCTURING ELEMENTS

A structuring element is a mini shape, by means of which image will be processed. It can be square, Rhombus, Disk, Linear, Triangle or any shape. Depending upon the requirement, the S.E. shape and size will be defined. Normally larger size of S.E suffer with performance degradation. So, it has to be designed by suitable smaller size of S.E's. So, these smaller S.E's will be applied on the image, as a series. [i.e. they will be applied iteratively on image]. The division of a S.E into a series or asset of mini S.E's is called S.E de-composition.

Normally, pattern restoration, will be done, by median filter, applying it iteratively. Its performance is good for image restoration, but the computational complexity is (very) high. But the equivalent impact may be obtained through morphological operations. In these morphological operations, the implementation involves less computational complexity. Some of the morphological operations are idempotent. So, choosing suitable morphological operation, these pattern restoration algorithms may be implemented. But very critical point is design of a suitable structuring element, which is optimal. It is discussed by DAN SCHONFELD (19). He discussed in his paper with in depth analysis, and suitable examples.

JIANNING XU has proposed a method for implementation (21) of S.E's on parallel computers, after decomposition of S.E's into a series of S.E's. Octavia I CAMPS & others (22) proposed algorithms of extension of S.E's and decomposition to gray - scale. GIOVANNI ANELLI & others (23) proposed algorithms for decomposition of arbitrarily shaped S.E's into a series, using genetic algorithms. They have done good experimentation also.

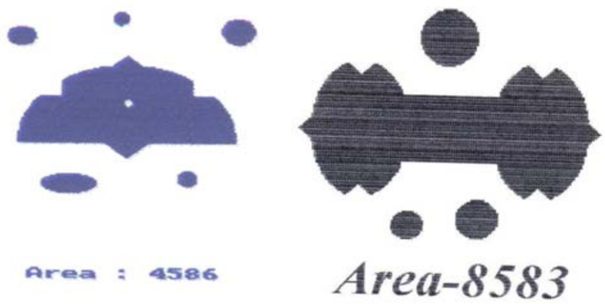
PITAL & VENETS ANOPOULOS also proposed methods for decomposition of S.E's, using iterative morphology and multi scale environment (20). To reduce the computational complexity, the S.E size should be minimum. So, optimization of points of S. E is discussed (24) by CRAIG H. RICHARDSON & RONALD W. SCHAFCR and optimal S.E is obtained by them, by their algorithms. HOCHONG PARIC & ROLAND T. CHIN (25) also proposed another method for S.E decomposition in optimal way, suitable to be implemented on parallel processors.

RONALD HOES & IMAANTS SVALBE extended the concept of S.E decomposition to gray scale, in iterative morphological environment (26). HOCHONG PARIC & RALAND T.CHIN also proposed algorithms (27) for S.E decomposition. They have given good mathematical basic back ground. P. SUSSNER, G.X. RITTER also discussed S.E. decomposition, but in gray – scale, using matrix algebra & other algebraic methods (28). A new methodology is discussed in this paper.

NINA. S. T. HIRATA (29) also contributed good research in M.M by developing algorithms for S.E decomposition. He has given the necessary back ground, and done good experimentation, and shown that, the results are excellent.

V. IMAGES AND OUTPUTS

The importance of multi scale morphology is explained in the section 3 . So,the primitive operations of mathematical morphology--erosion and dilation are taken and they are discussed in multi scale morphology point of view.The concept is very simple.But some of the important points are eloborated practically with the help of a few images and some important observations are given.



Semi-circle dumbbell

In this section, the results of experiments are presented. Actually two diagrams are taken, a Semi circle shape and a dumbbell shape. On these images various morphological operations are applied. The output is got in the form of tables, diagrams and graphs, around 1000 pages. But here some important as well as sample outputs are presented , relevant to this work.

IMAGES: In this section, the Semi circle and dumbbell images are presented.

OUTPUTS:

In this sub section, the results of **semi circle** are given area wise in multi scale as well as iterative environment, applying erosion and dilation. First table is of dilation and the second table is of erosion.

Areas for Dilated Images

Iteration	Area						
	Window Sizes						
	3/3	5/5	7/7	9/9	11/11	13/13	15/15
1	5210	5870	6576	7330	8132	8982	9880
2	5870	7330	8982	10807	12670	14297	15890
3	6576	8982	11759	14297	16693	19041	21136
4	7330	10807	14297	17501	20446	23094	25585
5	8132	12670	16693	20446	23753	26882	29632
6	8982	14297	19041	23094	26949	30352	32615
7	9880	15890	21136	25768	29898	32979	34089
8	10807	17501	23094	28274	32491	34585	34491
9	11759	19041	25089	30622	34487	35104	34596
10	12670	20446	27081	32766	35470	35308	34596

Areas for Eroded Images

Iteration	Area						
	Window Sizes						
	3/3	5/5	7/7	9/9	11/11	13/13	15/15
1	4004	3469	3000	2588	2223	1896	1580
2	3469	2588	1896	1269	684	267	40
3	3000	1896	964	267	0	0	0
4	2588	1269	267	0	0	0	0
5	2223	684	0	0	0	0	0
6	1896	267	0	0	0	0	0
7	1580	40	0	0	0	0	0
8	1269	0	0	0	0	0	0
9	964	0	0	0	0	0	0
10	684	0	0	0	0	0	0

The erosion and dilation operations are applied in multi scale environment on semi circle and dumbbell images. In this paper the results applied on **semi circle** are only presented. **The conclusions and observations are talied with dumbbell image also.**

6. Discussions:(GENERAL)

Normally according to structural element decomposition, a morphological operation will be equal to another iterative morphological operation, with smaller window size. From the outputs of semi-circle and dumbbell the above concept is studied in detail and some of the equalities are studied and recorded. In this paper the results corresponding to semi circle are presented. Basing upon this,some of the observations are done .These observations are taliedwith dumbbell image also ,which are available with author's data base. From these some of the observations are…

Erosion at 5/5 window is equivalent to erosion twice with 3/3 window size, on an image.

Erosion at 7/7 window size is equivalent to erosion thrice with 3/3 window size, on an image.

Erosion at 9/9 window size is equivalent to erosion four times, with 3/3 window size, on an image or erosion twice, with 5/5 window size, on an image.

Erosion at 15/15 window size is equivalent to erosion seven times, with 3 /3 window size, on an image.

In the same way, Erosion at 9/9 window size, twice, is equivalent to erosion four times, with 5/5 window size, on an image or erosion, with 3/3 window size, eight times on an image.

In the same way, Erosion twice with window size 11/11 is equivalent to erosion ten times, with window size 3/3 on an image.

In the same way, dilation at 5/5 window is equivalent to dilation twice with 3/3 window size, on an image.

In the same way, dilation at 7/7 window size is equivalent to dilation thrice with 3/3 window size, on an image.

Dilation at 9/9 window size is equivalent to dilation at 5/5 window size, twice, is equivalent to dilation at 3/3 window size, four times.

Dilation at 11/11 window size is equivalent to dilation 3/3 window size, five times.

Dilation at 13/13 window size is equivalent to dilation at 3/3 window size, six times.

Dilation at 13/13 window size is equivalent to dilation at 5/5 window size, thrice.

Dilation at 13/13 window size is equivalent to dilation at 7/7 window size, thrice.

Dilation at 15/15 window size is equivalent to dilation at 3/3window size, seven times.

Dilation at 9/9 window size, twice, is equivalent to dilation at 5/5 window size, four times, is equivalent to dilation at 3/3 window size eight times.

Dilation at 7/7 window size, thrice, is equivalent to dilation at 3/3 window size, nine times.

Dilation at 11/11 window size, twice, is equivalent to dilation at 5/5 window size, five times, is equivalent to dilation at 3/3 window size, ten times.

7 DISCUSSIONS:(EQUALITIES):

7.1 EQUALITIES EROSION: Equalities among iterative erosion.. The following Equalities are observed on results obtained on semi circle. **These are talied with results of dumbbell image also.**

$[E \text{ (at 3/3 window size)}]^2$ = [E (at 5/5 window size)] = 3469

$[E \text{ (at 3/3 window size)}]^3$ = [E (at 7/7 window size)] = 3000

$[E \text{ (at 3/3 window size)}]^4$ = $[E \text{ (at 5/5 window size)}]^2$= [E (at 9/9 window size)] = 2588

$[E \text{ (at 3/3 window size)}]^5$ = [E (at 11/11 window size)] = 2223

$[E \text{ (at 3/3 window size)}]^6$ = $[E \text{ (at 5/5 window size)}]^3$ =

$[E \text{ (at 7/7 window size)}]^2$= [E (at 13/13 window size)] = 1896

$[E \text{ (at 3/3 window size)}]^7$ = [E (at 15/15 window size)] = 1580

$[E \text{ (at 3/3 window size)}]^8$ = $[E \text{ (at 5/5 window size)}]^4$ = $[E \text{ (at 9/9 window size)}]^2$ = 1269

$[E \text{ (at 3/3 window size)}]^9$ = $[E \text{ (at 7/7 window size)}]^3$ = 964

$[E \text{ (at 3/3 window size)}]^{10}$ = $[E \text{ (at 5/5 window size)}]^5$ =

$[E \text{ (at 11/11 window size)}]^2$ = 684

7.2 EQUALITIES DILATION:

Equalities among iterative dilation..

$[D \text{ (at 3/3 window size)}]^2$ = [D (at 5/5 window size)] = 5870

$[D \text{ (at 3/3 window size)}]^3$ = [D (at 7/7 window size)] = 6576

$[D \text{ (at 3/3 window size)}]^4$ = $[D \text{ (at 5/5 window size)}]^2$= [D (at 9/9 window size)] = 7330

$[D \text{ (at 3/3 window size)}]^5$ = [D(at 11/11 window size)] = 8132

$[D \text{ (at 3/3 window size)}]^6$ = $[D \text{ (at 5/5 window size)}]^3$ = $[D \text{ (at 7/7 window size)}]^2$ =

[D (at 13/13 window size)] = 8982

$[D \text{ (at 3/3 window size)}]^7$ = [D (at 15/15 window size)] = 9880

$[D \text{ (at 3/3 window size)}]^8$ = $[D \text{ (at 5/5 window size)}]^4$ =

$[D \text{ (at 9/9 window size)}]^2$ = 10807

$[D \text{ (at 3/3 window size)}]^9$ = $[D \text{ (at 7/7 window size)}]^3$ = 11759

$[D \text{ (at 3/3 window size)}]^{10}$ = $[D \text{ (at 5/5 window size)}]^5$ =

$[D \text{ (at 11/11 window size)}]^2$ = 12670

8. FORMULAE:

In the following sub sections,formulae are given for erosion and dilation in multi scale as well as iterative environment,by the above discussions.

8 .1 <u>FORMULAE</u>: (Related To Erosion)

1. [E (at 3/3 window size)]^2 = [E (at 5/5 window size)]

2. [E (at 3/3 window size)]^3 = [E (at 7/7 window size)]

3. [E (at 3/3 window size)]^4 = [E (at 9/9 window size)]

4. [E (at 3/3 window size)]^5 = [E (at 11/11 window size)]

5. [E (at 3/3 window size)]^6 = [E (at 13/13 window size)]

6. [E (at 3/3 window size)]^7 = [E (at 15/15 window size)]

7 [E (at 5/5 window size)]^2 = [E (at 9/9 window size)]

8 [E (at 5/5 window size)]^3= [E (at 13/13 window size)]

9 [E (at 5/5 window size)]^1 = [E (at 3/3 window size)]^2

10 [E (at 5/5 window size)]^2 = [E (at 3/3 window size)]^4

11 [E (at 5/5 window size)]^3 = [E (at 3/3 window size)]^6

12 [E (at 5/5 window size)]^4 = [E (at 3/3 window size)]^8

13 [E (at 5/5 window size)]^5 = [E (at 3/3 window size)]^{10}

14 [E (at 7/7 window size)]^2 = [E (at 13/13 window size)]^1

15 E (at 7/7 window size)]^1 = [E (at 3/3 window size)]^3

16 E (at 7/7 window size)]^2 = [E (at 3/3 window size)]^6

17 E (at 7/7 window size)]^3 = [E (at 3/3 window size)]^9

18 E (at 9/9 window size)]^1 = [E (at 3/3 window size)]^4

19 E (at 9/9 window size)]^2 = [E (at 3/3 window size)]^8

20 E (at 11/11 window size)]^1 = [E (at 3/3 window size)]^5

21 E (at 11/11 window size)]^2 = [E (at 3/3 window size)]^{10}

22 E (at 13/13 window size)]^1 = [E (at 3/3 window size)]^6

8.2 FORMULAE (Related To dilation)

23 [D (at 3/3 window size)]^2 = [D (at 5/5 window size)]

24 [D (at 3/3 window size)]^3 = [D (at 7/7 window size)]

25 [D (at 3/3 window size)]^4 = [D (at 9/9 window size)]

26 [D (at 3/3 window size)]^5 = [D (at 11/11 window size)]

27 [D (at 3/3 window size)]^6 = [D (at 13/13 window size)]

28 [D (at 3/3 window size)]^7 = [D (at 15/15 window size)]

29 [D (at 5/5 window size)]^1 = [D (at 5/5 window size)]

30 [D (at 5/5 window size)]^2 = [D (at 9/9 window size)]

31 [D (at 5/5 window size)]^3= [D (at 13/13 window size)]

32 [D (at 5/5 window size)]^1 = [D (at 3/3 window size)]^2

33 [D (at 5/5 window size)]^2 = [D (at 3/3 window size)]^4

34 [D (at 5/5 window size)]^3 = [D (at 3/3 window size)]^6

35 [D (at 5/5 window size)]^4= [D (at 3/3 window size)]^8

36 [D (at 5/5 window size)]^5 = [D (at 3/3 window size)]^{10}

37 [D (at 7/7 window size)]^2 = [D (at 13/13 window size)]^1

38 D (at 7/7 window size)]^1 = [D (at 3/3 window size)]^3

39 D (at 7/7 window size)]^2 = [D (at 3/3 window size)]^6

40 D (at 7/7 window size)]^3 = [D (at 3/3 window size)]^9

41 D (at 9/9 window size)]^1 = [D (at 3/3 window size)]^4

42 D (at 9/9 window size)]^2 = [D (at 3/3 window size)]^8

43 D (at 11/11 window size)]^1 = [D (at 3/3 window size)]^5

44 D (at 11/11 window size)]^2 = [D (at 3/3 window size)]^{10}

45 D (at 13/13 window size)]^1 = [D (at 3/3 window size)]^6

9.DISCUSSIONS: FORMULAE

The above formulae can be interpreted in two ways.

In one way ,by means of equality. **The other way is S.E. decomposition**.

formula 1 can be interpreted as applying erosion by 5/5 S.E on image once is equal to applying 3/3 S.E twice,on the same image.

it can be explained in a different way also.

5/5 S.E can be decomposed to two structuring elements,having dim 3/3.

In the same way

formula 2 can be interpreted as applying erosion by 7/7 S.E on image once is equal to applying 3/3 S.E thrice ,on the same image.

it can be explained in a different way also

7/7 S.E can be decomposed to three structuring elements,having dim 3/3.

formula 4 can be interpreted as applying erosion by 11/11 S.E on image once is equal to applying 3/3 S.E five times ,on the same image.

it can be explained in a different way also.

11/11 S.E can be decomposed to five structuring elements,having dim 3/3.

formula 5 can be interpreted as applying erosion by 13/13 S.E on image once is equal to applying 3/3 S.E six times ,on the same image.

it can be explained in a different way also

13/13 S.E can be decomposed to six structuring elements , having dim 3/3

formula 23 can be interpreted as applying dilation by 5/5 S.E on image once is equal to applying 3/3 S.E twice ,on the same image.

it can be explained in a different way also.

5/5 S.E can be decomposed to two structuring elements , having dim 3/3.

formula 26 can be interpreted as applying dilation by 11/11 S.E on image once is equal to applying 3/3 S.E five times ,on the same image.

it can be explained in a different way also.

11/11 S.E can be decomposed to five structuring elements , having dim 3/3.

formula 31 can be interpreted as applying dilation by 13/13 S.E on image once is equal to applying 5/5 S.E three times ,on the same image.

it can be explained in a different way also.

13/13 S.E can be decomposed to three structuring elements , having dim 5/5.

10. CONCLUSIONS:

In this paper erosion and dilation are studied with reference to size of the structuring elrments in multi scale as well as iterative environments.The equalities are established in multi scale as well as iterative environments.

This paper will give discussions on fundamental characteristics of mathematical morphology in a different way. This type of practical discussion on examples is not done till now. Formulae are also given in this iterative and multi scale environment. This can be said as detailed prctical study in a narrow area in new environment,with out put formulae.This will lead to good understanding of the area as well as good applications, in to various areas.The above formulae will provide researchers new thoughts of applications to new areas as well as expansion of this mathematical morphology area.

This paper will give idea of structuring element de composition also up to some extant, practically.

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